Acceleration of positrons by electron beam-driven wakefields in a plasma

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Plasma wakefield acceleration of positron beams in the wake of a dense electron beam (in the blowout regime) is numerically analyzed. The acceleration is possible only if the energy content of the wakefield is not very high. This is in contrast to electron acceleration, for which the optimum performance requires driver currents and wave energies to be as high as possible. For positrons, the efficiency of plasma-to-witness energy exchange can amount to several tens percent, but high efficiencies require precise location of the positron beam and sophisticated beam shapes. Unlike an electron witness, the positron always gets an energy spread of about several percent caused by the transverse inhomogeneity of the accelerating field. © 2007 American Institute of Physics.

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I. INTRODUCTION

Plasmas can sustain electric fields that are many orders of magnitude higher than those in conventional accelerating structures. This property is used in plasma wakefield accelerators (PWFA), in which one particle beam drives the high amplitude field in the plasma, and another beam (witness) is accelerated by this field.\textsuperscript{1–4}

For collider applications of PWFA, it is desirable to succeed in acceleration of both electrons and positrons in the plasma. Research on electron acceleration received the major attention and already got crowned by understanding of the physics involved and by experimental demonstration of high energy gains.\textsuperscript{5} Studies of positron acceleration were mainly concept\textsuperscript{12} and as yet limited to positron-driven waves.\textsuperscript{13–15} Positrons, however, are rather “expensive” particles to be used as a driver in other conceivable applications of PWFA. Therefore it is important to analyze the possibility of positron acceleration in the wake of an electron beam.

In the linear regime, when densities of the beams are much lower than the plasma density, there is no principal difference in behavior of electrons and positrons. The difference appears at higher beam densities and nonlinear plasma responses. If the density of the electron driver is greater than the plasma density, then PWFA switches to the so-called blowout regime.\textsuperscript{16} In this regime, all plasma electrons are ejected off the beam propagation channel, and an electron-free region is formed around the drive beam. This region is called bubble,\textsuperscript{17} ion channel,\textsuperscript{16} or cavern;\textsuperscript{18} we will use the first term in what follows. In the case of electron acceleration, the blowout regime offers several advantages over the linear regime: linear focusing fields and radially independent accelerating fields inside the bubble\textsuperscript{16} (if the beam radius is not too small\textsuperscript{19}), the highest acceleration rate at a given plasma density, and a high efficiency.\textsuperscript{20} In this paper we check the option of positron acceleration in the blowout wake of a dense electron beam.

All simulations presented in the paper are made with the two-dimensional hybrid code LCODE (Refs. 21 and 22) in the axisymmetric geometry \((r, \varphi, z)\) with the \(z\)-axis as the direction of the beam propagation. For characterization of beam-plasma energy exchange and wakefield energy content, we use the dimensionless energy flux in the co-moving window:\textsuperscript{18}

\[
\tilde{\Psi} = \frac{4\pi e^2}{mc^2} \int_0^\infty \left( \frac{c}{8\pi} ((E_r - B_\varphi)^2 + E_z^2) + \sum_{u.v.} (\gamma - 1) mc^2 (c - v_u) \right) \times 2\pi dr,
\]

where the summation is carried out over plasma electrons in the unit volume, \(\gamma\) is the relativistic factor of plasma electrons, \(v_u\) is their longitudinal velocity, \(m\) is the electron mass, \(e\) is the elementary charge \((e > 0)\), \(c\) is the speed of light, and notation for the fields is common. Focusing properties of the wave are conveniently illustrated by maps of the dimensionless potential \(\tilde{\Phi}\), derivatives of which show the force exerted on axially propagating ultrarelativistic positron:

\[
\frac{\partial \tilde{\Phi}}{\partial z} = -\frac{\omega_p E_z}{cE_0}, \quad \frac{\partial \tilde{\Phi}}{\partial r} = -\frac{\omega_p}{cE_0} (E_r - B_\varphi),
\]

where \(\omega_p = \sqrt{4\pi ne^2/m}\) is the plasma frequency, \(E_0 = mce_0/\gamma\) is the unperturbed plasma density. For hydrodynamically responding plasmas, \(\tilde{\Phi} = \gamma(1-v_u/c)\). As the wave breaks, the simple relation between \(\tilde{\Phi}\) and the average plasma velocity disappears.

In Sec. II we analyze the transverse equilibrium of positrons and some of beam loading issues. We show that stable monoenergetic acceleration is possible only at a moderate energy content of the nonlinear plasma wave. In Sec. III we numerically determine the size of the unloaded accelerating basket as a function of the energy flux \(\tilde{\Psi}\), draw the efficiency map for energy takeoff, and find the witness shapes that minimize the energy spread for a given energy content of the wave. The main findings are summarized in Sec. IV.

II. TRANSVERSE EQUILIBRIUM AND BEAM LOADING

The first problem specific to positrons is related to the transverse equilibrium. As the energy content of the plasma wave increases, the region of favorable (inward) focusing

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force for positrons shrinks. In the blowout regime, in the limit of \( \bar{\Psi} \approx 1 \) the length of this region tends to zero [Figs. 1(a) and 1(c)]. When initially placed on axis in the accelerating field [at the head of the second bubble [Fig. 1(b)], positrons shift radially and take up an off-axis position [Fig. 1(c)], resulting in a drastic growth of the beam emittance.

There are, at least, two ways to improve the transverse  

FIG. 1. (Color online) (a) Bubble shape at a high energy content of the wave (map of the plasma electron density at \( \bar{\Psi}=70 \)); (b) corresponding profiles of the on-axis electric field and driver density; (c) contour lines of the force potential \( \bar{\Phi} \) showing the shape of the potential well near the end of the first bubble [in the area marked in (a) by the gray rectangle]. The thick line in (c) shows the local minima of \( \bar{\Phi} \) at \( z=\text{const} \); gray shading is the equilibrium location of relativistic test particles initially placed near the axis.

FIG. 2. Bubble shapes (a) at a high wave energy content in the presence of a rectangular channel (\( \bar{\Psi}=55 \), the same beam as in Fig. 1) and (c) at a low wave energy content in the uniform plasma (\( \bar{\Psi}=0.3 \)); [(b) and (d)] corresponding shapes of potential wells near regions of positron acceleration [location of the regions are shown by rectangles in (a) and (c)]. Gray shadings in (b) and (d) mark areas of both acceleration and favorable focusing.
field structure in the blowout regime. The first one is to make an ion-free channel on axis. With no ions, there is no source of the transverse force near the bubble axis, and positrons stay in indifferent transverse equilibrium. For example, in Figs. 2(a) and 2(b) the bubble shape and isolines of the force potential \( \Phi \) for the rectangular density channel

\[
n = \begin{cases} 
0, & r < c/\omega_p, \\
n_1, & r \geq c/\omega_p 
\end{cases}
\]

are shown.

Another way is to operate at moderate energy contents of the plasma wave. In this case, there exists a lengthy interval of high electron density between the first and second bubble [Fig. 2(c)]. The potential well in this interval has the minimum on axis [Fig. 2(d)], which means the favorable focusing.

The second problem of positron acceleration is related to the slope of the accelerating field. A dense witness always reduces the field as compared to the unloaded case. The difference between the loaded and unloaded fields increases towards the end of the witness (Fig. 3). If the derivative \( \partial E_z / \partial z \) is positive even in the unloaded case, then the witness augments it further, and zero energy spread \( (\partial E_z / \partial z = 0) \) cannot be achieved at any witness shape. For an electron witness, this problem does not appear since \( \partial E_z / \partial z < 0 \) in the favorable region [Fig. 1(b)]. For positrons, the interval of \( \partial E_z / \partial z < 0 \) at high energy contents of the wave is very short [Fig. 1(b)], and the presence of a density channel does not improve the situation (Fig. 3). Thus, unlike electrons, for positrons the optimum performance is not associated with maximization of the peak driver current and wave energy.

### III. POTENTIALITIES OF POSITRON ACCELERATION

As we see, the optimum place for accelerated positrons is located between the first and the second bubble. Let us characterize the size and location of this region (acceleration basket) by the length of the favorable field slope \( \Delta \xi_p \) and the offset with respect to the first bubble \( \Delta \xi_0 \) (Fig. 4). The third important dimension is the length of the favorable focusing interval \( \Delta \xi_f \) [Fig. 2(d)]; it is also measured from the end of the first bubble. The measure of the field strength is the maximum field \( E_m \) (Fig. 4).

At first glance it would seem that the above quantities depend on many driver parameters like charge, radius, length, and shape. In reality, once the bubble is formed, the wave characteristics over a wide range of driver parameters are well determined by the single number, namely, the energy flux \( \Psi \) behind the driver. The flux dependence of the maximum field is shown in Fig. 5. Here the dots correspond to Gaussian-shape drivers of various rms length \( \sigma_z \) \((0.02 \leq \sigma_z/c < 3)\) and peak current \( I_m \) \([0.1 \leq I_m e/(mc^2) \approx 1.2]\).

Analogous points for the rectangular \( z \) distribution of the

![Fig. 3. Electric field and beam density on axis for the same driver and plasma as in Figs. 2(a) and 2(b) with and without a dense positron witness.](image)

![Fig. 4. Electric field and beam density on axis for the same driver as in Figs. 2(c) and 2(d).](image)

![Fig. 5. (Color online) Maximum accelerating field \( E_m \) as a function of the dimensionless energy flux \( \Psi \).](image)

![Fig. 6. (Color online) Spatial dimensions of the acceleration basket as functions of the dimensionless energy flux \( \Psi \) in normal (a) and semilogarithmic (b) scales.](image)
driver current fit the same curve. At moderate $\tilde{\Psi}$, the maximum field is well approximated by the formula

$$E_m \approx 0.5E_0 \sqrt{\tilde{\Psi}}$$

(thin line in Fig. 5). At high $\tilde{\Psi}$, the field growth is close to the linear one. For longer beams ($\sigma_z\omega_p/c \geq 3$), the beam tail touches the bubble end and the parameters of the accelerating basket deviate from the above scaling.

Spatial dimensions of the acceleration basket are shown in Fig. 6. Two facts are seen from these graphs. First, $\Delta\xi_0 < \Delta\xi_f < \Delta\xi_m$, that is, the acceleration basket begins when the field $E_z$ becomes positive and ends as the focusing force changes its sign. Second, all sized exponentially decrease as the energy content of the wave grows. For example,

$$\Delta\xi_m = e^{-\tilde{\Psi}/3} c/\omega_p$$

(thin lines in Fig. 6). Thus, for high $\tilde{\Psi}$ it is extremely difficult to place accelerated positrons properly.

Once properties of the acceleration basket are determined by the dimensionless energy flux $\tilde{\Psi}$, it is natural to assume that the efficiency of positron acceleration is determined mainly by $\tilde{\Psi}$ and by the field inside the witness $E_w$. We will define the efficiency as the ratio of the energy taken by the witness to the energy left by the driver in a unit length of the plasma. The maximum population of the positron beam is found in the same manner as it was done in Ref. 20 for electron beams. Namely, the witness current is adjusted slice-by-slice towards decreasing $z$ in such a way as to have $E_z = E_w = \text{const}$ on the axis. The witness ends when $E_z$ gets smaller than $E_w$ or the radial force near the axis changes its sign. The efficiency map thus obtained remains qualitatively the same as the driver shape or the witness radius change.

In Fig. 7, the efficiency map is plotted for the Gaussian driver with $\sigma_z=1.4c/\omega_p$, $\sigma_r=0.1c/\omega_p$ and the same witness radius. The flux $\tilde{\Psi}$ varies by changing the driver current. With a fine enough simulation grid, it is possible to construct a matched witness for very high $\tilde{\Psi}$ and low $E_w$ (top left corner of Fig. 7), but the location of this witness must be controlled with exponentially high precision (see Fig. 6). We exclude these unrealistic variants from consideration and show only the cases for which the distance between the wit-
ness and the point of $E_z=0$ is greater than $0.01c/\omega_p$ (below the thick line in Fig. 7).

We see from Fig. 7 that plasma-to-witness efficiencies of several tens percent are possible for positron beams. The ultimate efficiency is limited by the precision of witness shaping. At higher dimensionless accelerating rates, higher energy content of the wave is required to stay within reasonable witness sizes.

Examples of matched driver-witness systems are shown in Fig. 8. The points corresponding to these examples are marked in Fig. 7 by crossed dots. We see from Fig. 8 that the positron beam is located in the region of high density of plasma electrons. As a consequence, the accelerating field varies across the beam even if on the axis it is exactly $E_u$. The resulting energy spread (of about several percent) cannot be minimized by reduction of the witness radius, since a narrow and dense positron beam attracts plasma electrons stronger and produces a higher inhomogeneity of $E_z$. The motion of plasma electrons in the presence of the witness is rather complicated, which determines a complicated shape of the matched witness. Plasma electrons attracted by the witness extend the potential well for positrons (see maps of $\Phi$ in Fig. 8), so defocusing is not a limiting factor for a dense witness.

IV. SUMMARY

As follows from the above simulations, it is possible to accelerate positrons in the blowout regime of the plasma wakefield accelerator, if the energy content of the wakefield is not very high. This contrasts with electron acceleration, for which the optimum performance is associated with maximization of the peak driver current and wave energy. The positron beam must be placed between the first and the second bubble, in the region of increased density of plasma electrons. The efficiency of plasma-to-witness energy exchange can be as high as several tens percent. High efficiencies require precise location of the positron beam and sophisticated beam shapes. Unlike electron witness, the positrons always get an energy spread of about several percent caused by the transverse inhomogeneity of the accelerating field.

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