On the Behavior of the Effective QCD Coupling $\alpha_\tau(s)$ at Low Scales

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hep-ph/0212078

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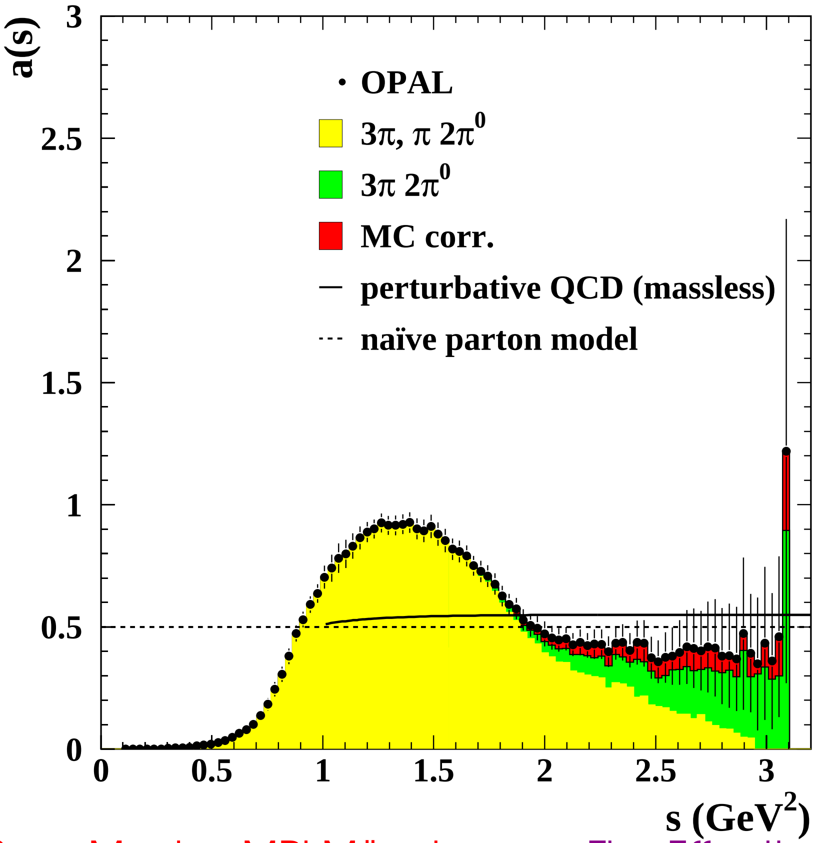
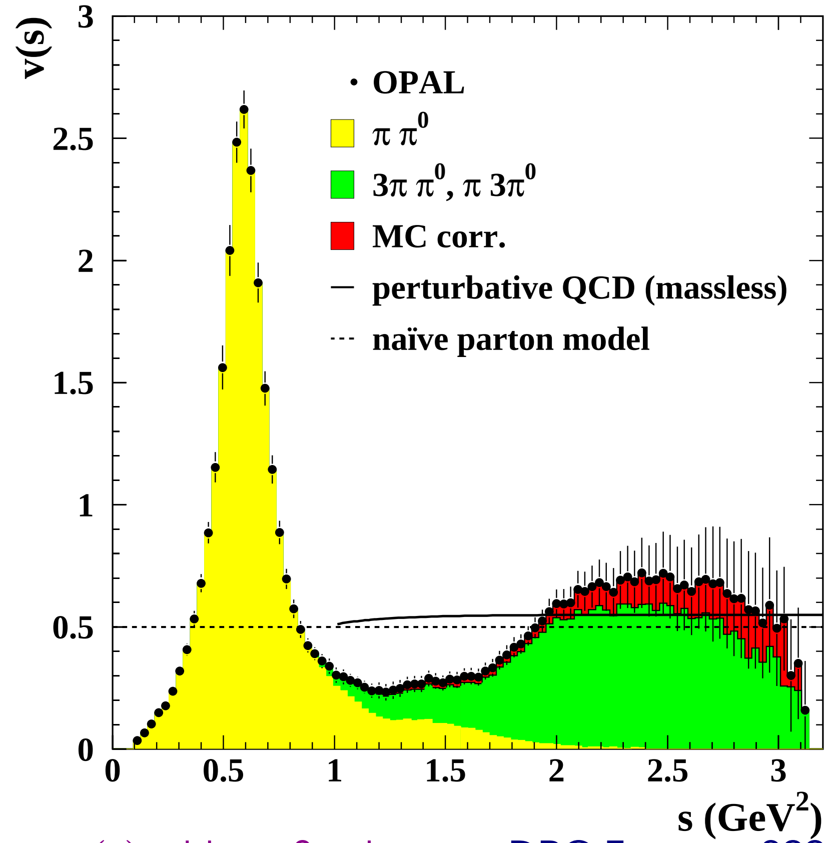
Motivation

- perturbative QCD is not well defined in the infrared domain
  - ▷ usually the $\overline{\mathrm{MS}}$-scheme is used to define the strong coupling constant
  - ▷ $\alpha_s^{\overline{\mathrm{MS}}}(s) \to \infty$ for $s \to \Lambda^2_{\overline{\mathrm{MS}}}$
  - ▷ this behavior makes the perturbative expansion of physical observables problematic (factorial growth of the coefficients)
- alternative procedure is to define a QCD coupling from a given physical observable
- these couplings are called effective charges
  - ▷ all order resummations of perturbation theory
  - ▷ include all non-perturbative effects
  - ▷ guaranteed to be analytic and non-singular
  - ▷ finite as s goes to 0
  - ▷ but also freezing to a constant value?

Hadronic τ Decays

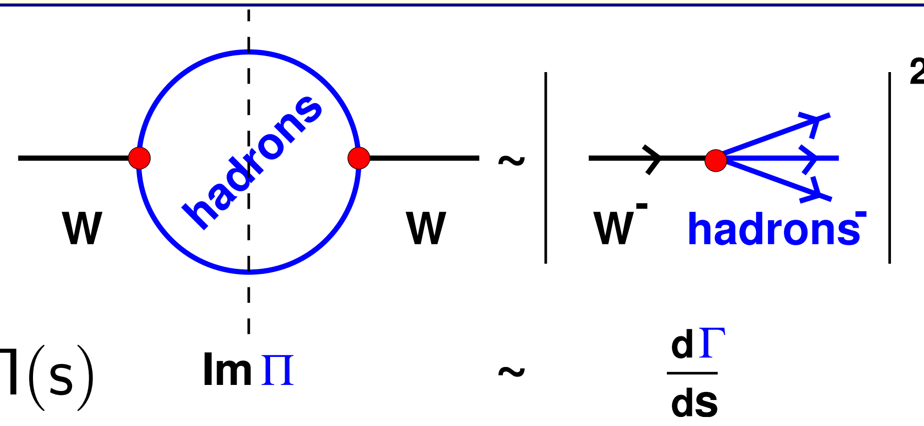
- hadronic τ decays are an ideal QCD laboratory
  - $R_\tau = \frac{\Gamma(\tau \to \text{hadrons}\,\nu_\tau)}{\Gamma(\tau \to e\,\nu_e\,\nu_\tau)} = 3\,S_{EW}\left(|V_{ud}|^2 + |V_{us}|^2\right)(1 + \delta_{pert} + \delta_{non-pert})$
  - for non-strange τ decays V and A current can be separated
  - mass effects are small
  - spectral functions are measured with excellent accuracy
- OPAL data (Eur. Phys. J. **C7**, (1999) 571.):



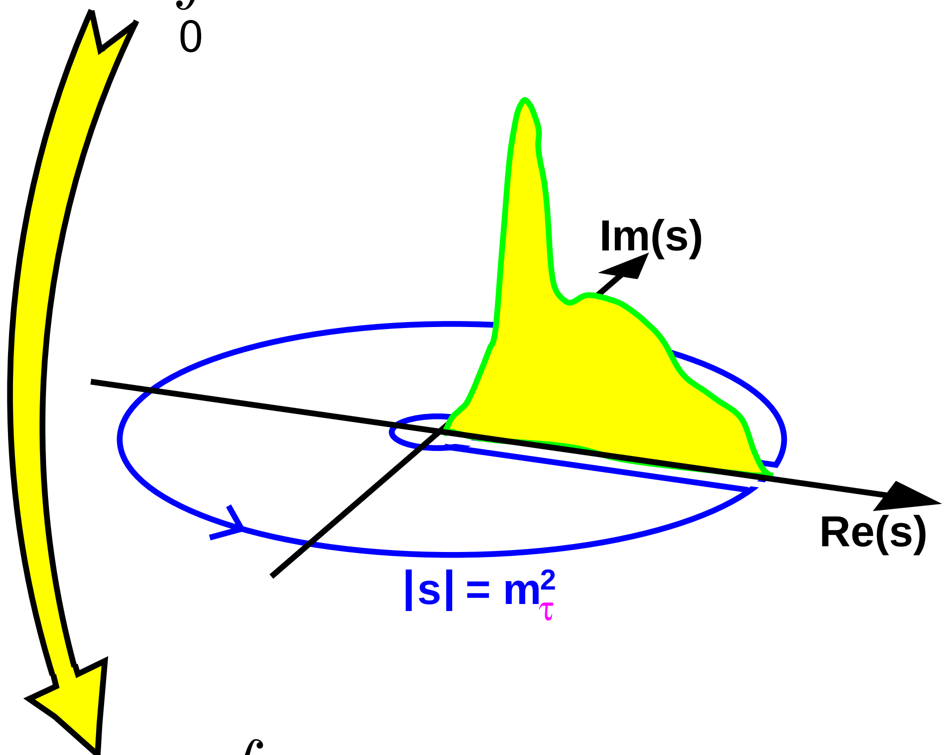


Hadronic τ Decays II

- Optical theorem:



- $R_\tau = 12\pi \int_0^{m_\tau^2} ds\, \mathrm{poly}(s/m_\tau^2)\, \mathrm{Im}\Pi(s)$



- Cauchy's theorem

- $R_\tau = 6\pi i \oint_{|s|=m_\tau^2} ds\, \mathrm{poly}(s/m_\tau^2)\, \Pi(s)$

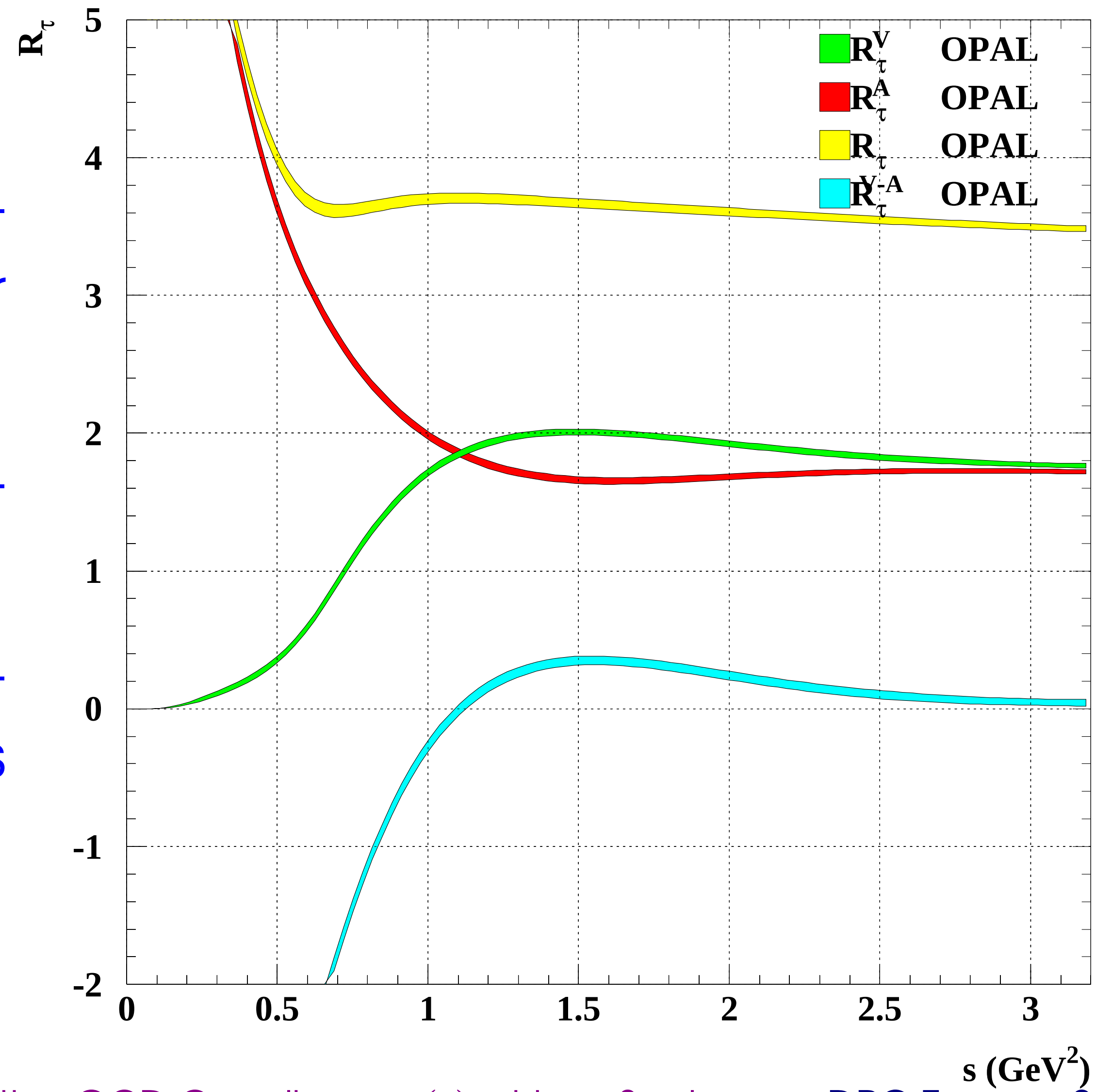
Decay of a Hypothetical τ' lepton

- 'Define' a hypothetical τ' lepton with mass $m_{\tau'} \leq m_\tau$
- $R_{\tau'} = 6\pi i \oint_{|s|=m'^2_\tau} ds\, \text{poly}(s/m^2_{\tau'})\, \text{Im}\Pi(s)$

▷ same endpoint suppression for τ' as for real τ

▷ $V - A$ has no perturbative contribution

▷ for $V + A$ most non-perturbative parts cancel out



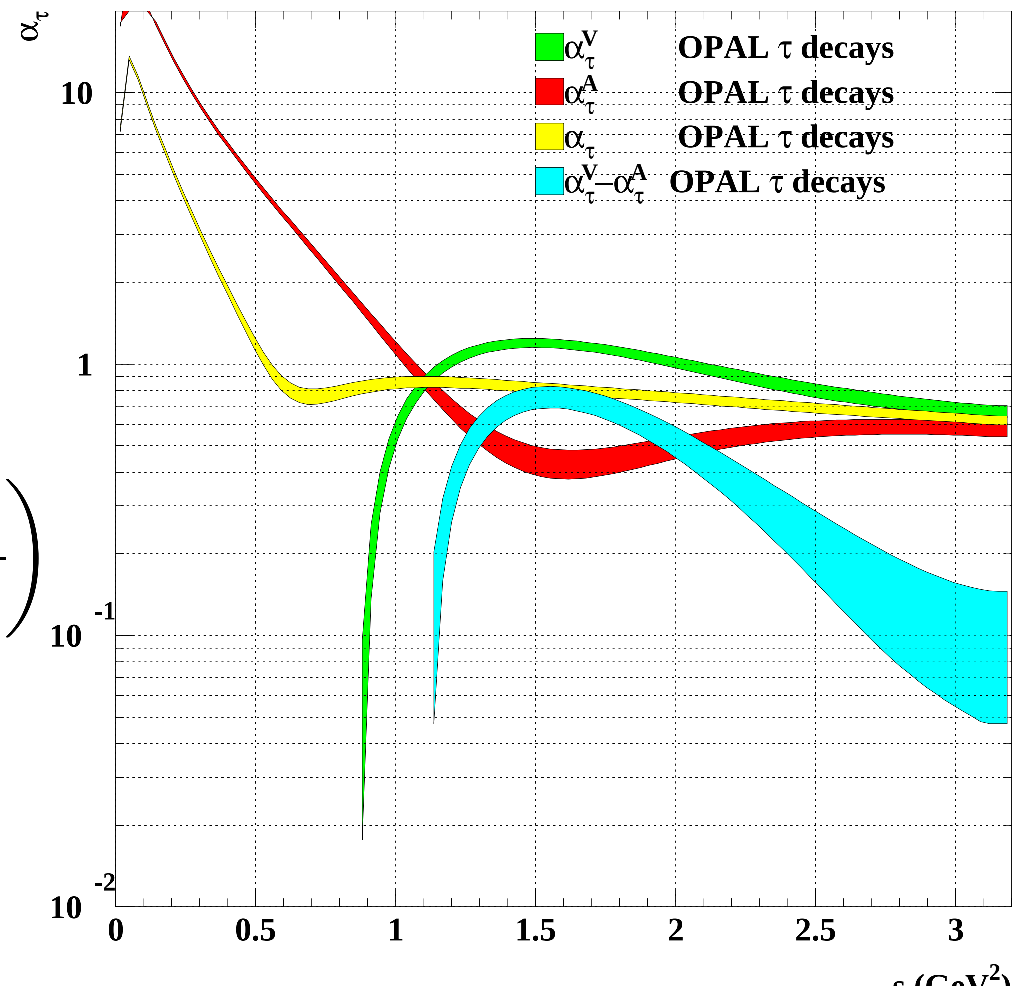
Effective Charge

- replace the usual power series in α_s
  - ▷ $R_{\tau'}^{V+A} =$

$$3\,S_{EW}\,|V_{ud}|^2\left(1+\frac{\alpha_s(m_{\tau'}^2)}{\pi}+5.20\frac{\alpha_s^2(m_{\tau'}^2)}{\pi^2}+26.4\frac{\alpha_s^3(m_{\tau'}^2)}{\pi^3}+\ldots\right)$$

- with effective charge:
  - ▷ $R_{\tau'}^{V+A} =$

$$3\,S_{EW}\,|V_{ud}|^2\left(1+\frac{\alpha_\tau(m_{\tau'}^2)}{\pi}\right)$$



Effective Charge II

- perturbative expansion of α_τ:

$$\triangleright\ \frac{\alpha_\tau(s)}{\pi} = \frac{\alpha_{\overline{MS}}(s)}{\pi} + \left(\frac{19}{48}\beta_0 + K_2\right)\frac{\alpha^2_{\overline{MS}}(s)}{\pi^2}$$

$$+\left\{\left[\frac{265}{1152} - \frac{1}{48}\pi^2\right]\beta_0^2 + \frac{19}{192}\beta_1 + \frac{19}{24}\beta_0 K_2 + K_3^{\overline{MS}}\right\}\frac{\alpha^3_{\overline{MS}}(s)}{\pi^3}$$

$$+\left\{\left[\frac{3355}{18432} - \frac{19}{768}\pi^2\right]\beta_0^3 + \left[\frac{1325}{9216} - \frac{5}{384}\pi^2\right]\beta_0\,\beta_1 + \frac{19}{768}\beta_2^{\overline{MS}}\right.$$

$$\left. + \left[\left(\frac{265}{384} - \frac{1}{16}\pi^2\right)\beta_0^2 + \frac{19}{96}\beta_1\right]K_2 + \frac{19}{16}\beta_0\,K_3^{\overline{MS}} + K_4^{\overline{MS}}\right\}\frac{\alpha^4_{\overline{MS}}(s)}{\pi^4}$$

- β function of α_τ:

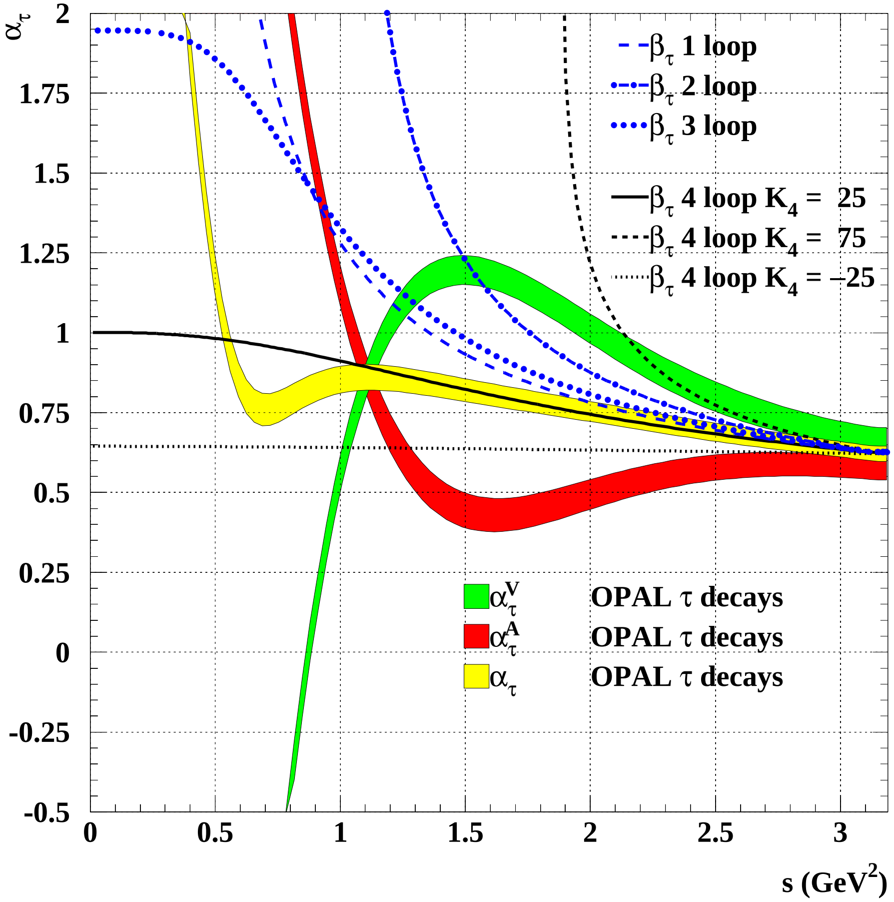
$$\triangleright\ \beta_{\tau,0} = \beta_0 = 9, \quad \beta_{\tau,1} = \beta_1 = 64$$

$$\beta_{\tau,2} = \frac{79813}{16} + 6552\,\zeta_3 + 5400\,\zeta_5 - 243\,\pi^2 - 11664\,\zeta_3^2 \simeq -788.504$$

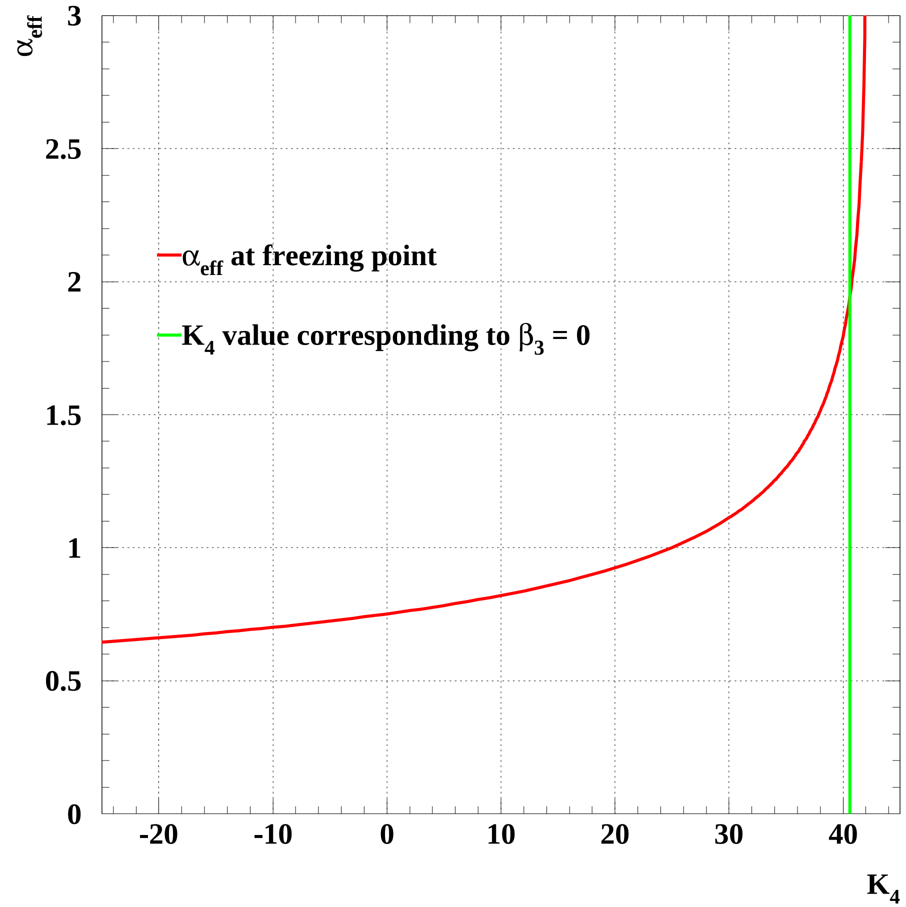
$$\beta_{\tau,3} = -\frac{585179735}{144} + 4820288\,\zeta_3 - 1614600\,\zeta_5 + 1166400\,\zeta_3\,\zeta_5$$

$$+1000512\,\zeta_3^2 - 8640\,\pi^2 - 1679616\,\zeta_3^3 + 1152\,K_4^{\overline{MS}} \simeq -46776.026 + 1152 K_4^{\overline{MS}}$$

Infrared Behavior



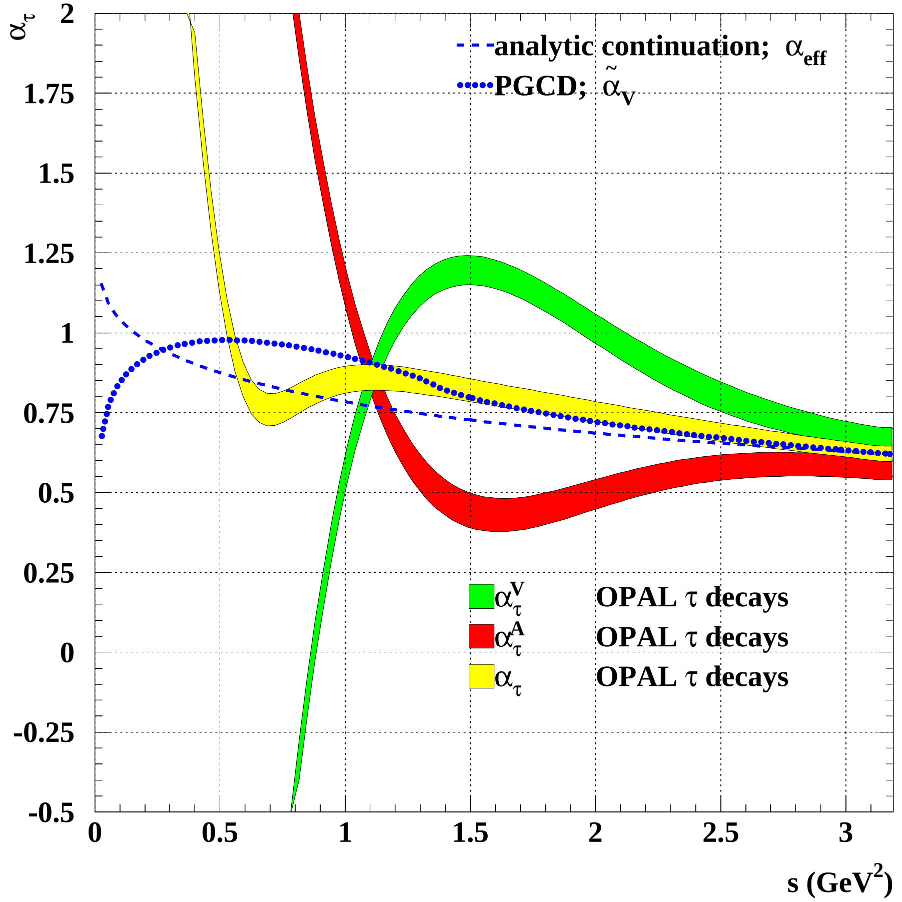
- β-functions with positive coefficients have singularity
- for β_τ the first non-universal coefficient is negative
  - ▷ α_τ freezes in the infrared



- $\beta_{\tau,3}$ depends on $K_4^{\overline{MS}}$
  - ▷ freezing occurs for $K_4 < 41.98$

Infrared Behavior II

- Comparison with other examples of freezing couplings:



▷ One-loop "timelike" coupling

$$\alpha_{eff}(s) = \frac{4\pi}{\beta_0}\left\{\frac{1}{2} - \frac{1}{\pi}\arctan\left[\frac{1}{\pi}\ln\frac{s}{\Lambda^2}\right]\right\},$$

obtained from the analytic continuation of

$$\alpha_s(Q^2) = 4\pi/\left[\beta_0 \ln(Q^2/\Lambda^2)\right],$$

which defines the spectral density

$$\frac{d\alpha_{eff}(s)}{d\ln s} = \frac{\alpha_s(-s+i\epsilon) - \alpha_s(-s-i\epsilon)}{2\pi i}.$$

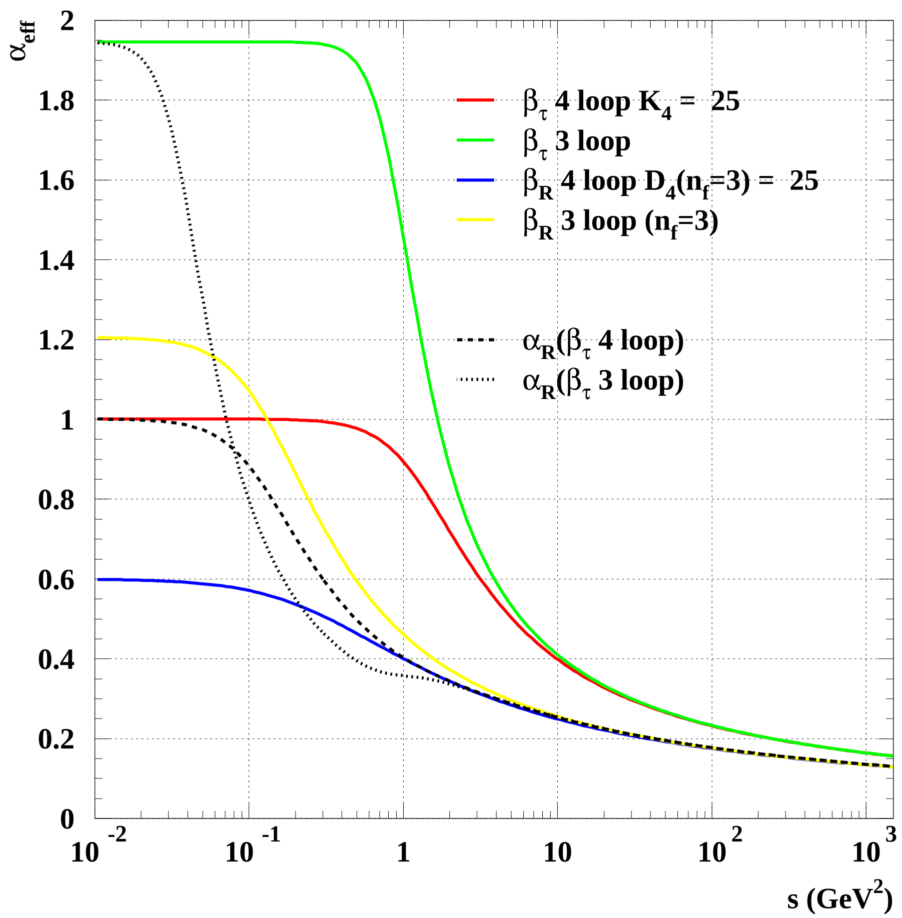
▷ this α_{eff} freezes to $4\pi/\beta_0$ as s goes to 0.

▷ modified $\tilde{\alpha}_V$ coupling calculated from the static quark potential using perturbative gluon condensate dynamics.

▷ both couplings normalized via Λ to match α_τ at m_τ^2

Relation to other Physical Observables

- effective couplings are linked via commensurate scale relations
- $\alpha_R(m^2_{\tau'}) = \alpha_\tau\left(m^2_{\tau'}\exp\left(\frac{19}{12} + \frac{169}{64}\frac{\alpha_\tau(m^2_{\tau'})}{\pi}\right)\right)$



- ▷ $\alpha_R \sim 0.85$ for $s < 0.1\,GeV^2$ according to Mattingly and Stevenson Phys. Rev. D **49**, 437 (1994)
- ▷ commensurate scale relations are consistent with this result

Conclusions

- defining the QCD coupling directly from physical observables is advantageous
  - ▷ resulting coupling stays finite in the infrared
  - ▷ is analytic
  - ▷ has no scheme or scale ambiguities
- α_τ has near constant behavior at low mass scales
- OPAL data is consistent with freezing of α_τ at mass scales $s \sim 1\,\mathrm{GeV}^2$ with a magnitude $\alpha_\tau \sim 0.9 \pm 0.1$
- other physical observables can be related to α_τ via commensurate scale relations
  - ▷ this result is compatible with the observed infrared freezing of α_R obtained by Mattingly and Stevenson
    Phys. Rev. D **49**, 437 (1994)