α_s

from τ decays

and e^+e^- annihilation

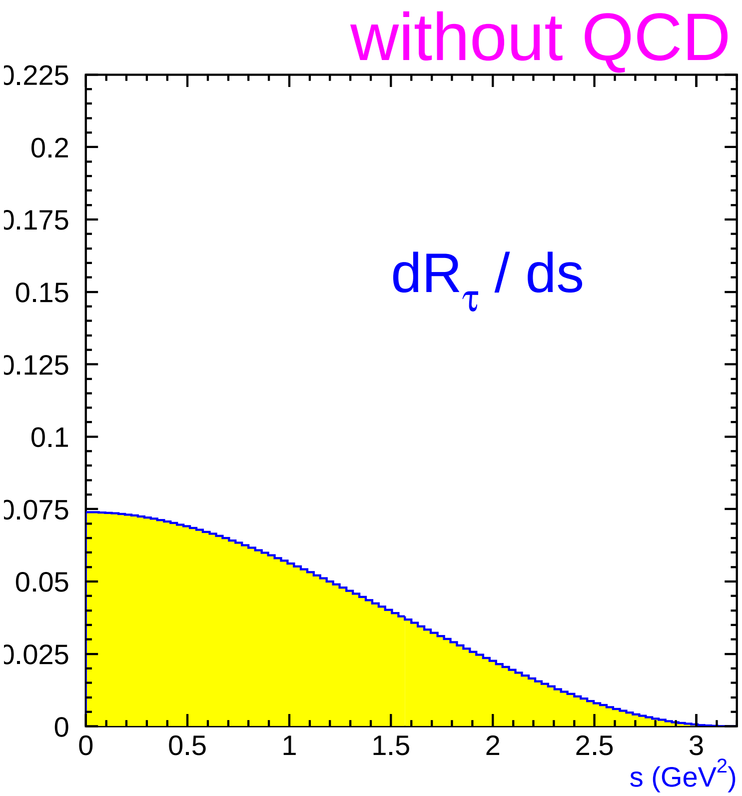
PEP-N Workshop 30. April 2001, SLAC Sven Menke, SLAC

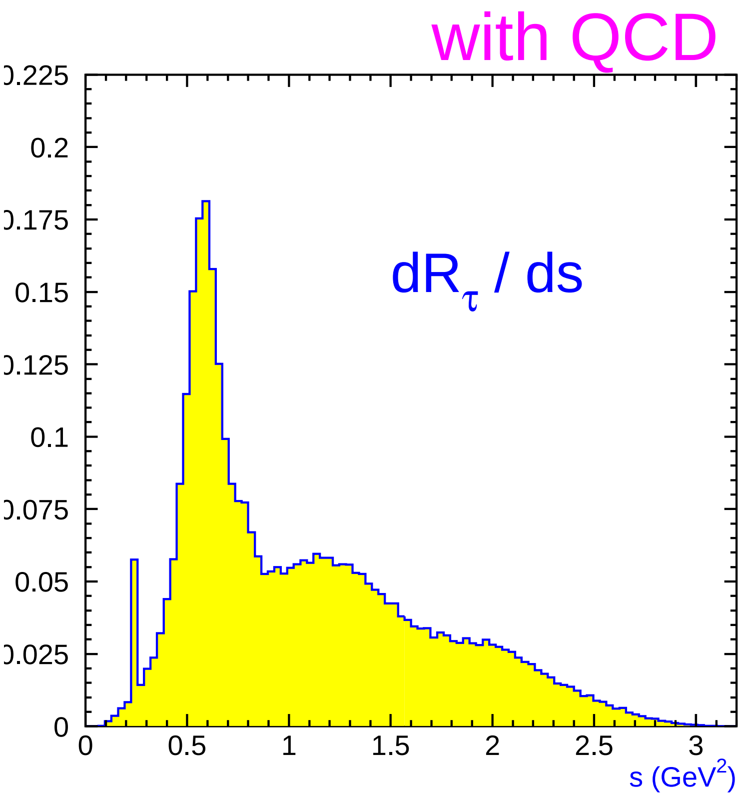
- R_τ and the strong coupling
  - spectral functions
  - perturbative and non-perturbative QCD
  - the role of moments
  - QCD fit results
  - hypothetical τ decays
- $R_{e^+e^-}$ and the strong coupling
  - combination of different experiments
  - $R_{e^+e^-}$ including new data up to $4.5\,\mathrm{GeV}^2$
  - why not use moments too?
  - preliminary QCD fit results
- Conclusion

The Hadronic Decay Ratio of the τ

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \text{ hadrons})}{\Gamma(\tau \to \nu_\tau \, e \, \nu_e)}$$

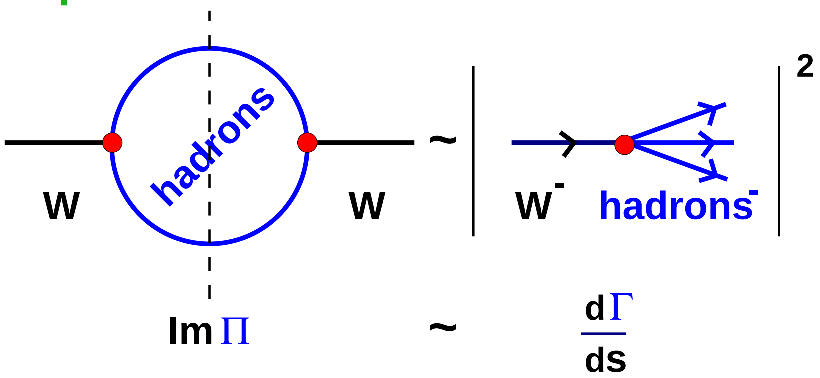
- Tree: $R_\tau = N_C \left(|V_{ud}|^2 + |V_{us}|^2\right) = 3$
- Exp.: $R_\tau = 3.65$





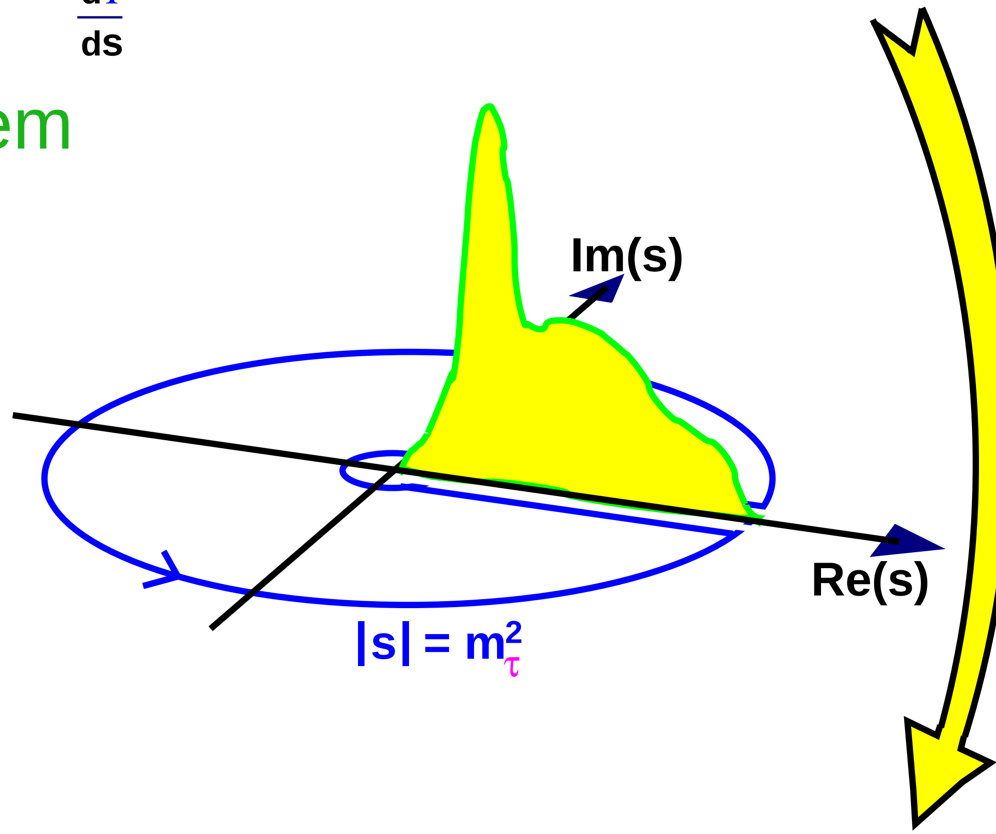
R_τ and the Spectral Functions

Optical theorem



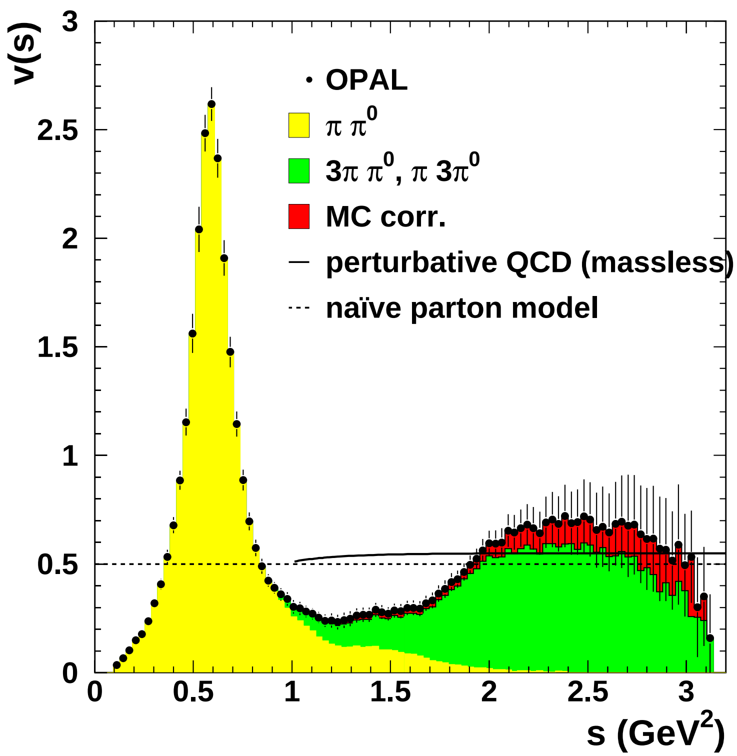
$$R_\tau = 12\pi \int_0^{m_\tau^2} ds\ \mathrm{poly}(s)\, \mathrm{Im}\Pi(s)$$

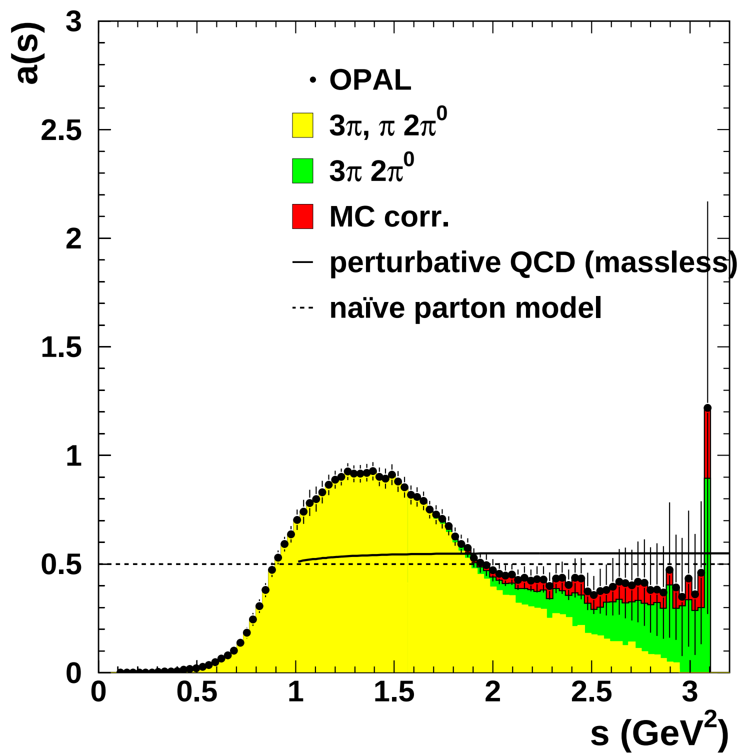
Cauchy's theorem



$$R_\tau = 6\pi i \oint_{|s|=m_\tau^2} ds\ \mathrm{poly}(s)\, \Pi(s)$$

Spectral Functions





$$v(s) \sim \mathrm{Im}\Pi_V^{(1)} \qquad a(s) \sim \mathrm{Im}\Pi_A^{(1)}$$

$$v/a(s) \sim \left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1} \sum_{h_{V/A}} \frac{B(\tau \to h_{V/A})}{B(\tau \to e)}\left(\frac{1}{N}\frac{dN}{ds}\right)_{V/A}$$

OPAL Collaboration, K. Ackerstaff et al., Eur. Phys. J. **C7** (1999) 571.

Theoretical Description of R_τ I

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW}\left(1 + \delta_{\text{pert}} + \delta^{V/A}_{\text{mass}} + \delta^{V/A}_{\text{non-pert}}\right)$$

- perturbative part

$$1+\delta_{\text{pert}} = \frac{1}{2\pi i}\oint_{|s|=m_\tau^2} \frac{ds}{s}\left(1-2\frac{s}{m_\tau^2}+2\frac{s^3}{m_\tau^6}-\frac{s^4}{m_\tau^8}\right)\underbrace{(-s)\frac{d\Pi}{ds}}_{D(s)}$$

CIPT: $D(s) \sim 1+\frac{\alpha_s(-s)}{\pi}+1.64\frac{\alpha_s^2(-s)}{\pi^2}+6.37\frac{\alpha_s^3(-s)}{\pi^3}$

FOPT: $1+\delta_{\text{pert}} = 1+\frac{\alpha_s(m_\tau^2)}{\pi}+5.20\frac{\alpha_s^2(m_\tau^2)}{\pi^2}+26.4\frac{\alpha_s^3(m_\tau^2)}{\pi^3}$

RCPT: $D(s) \sim 1+\sum_{n=1}^{\infty}\kappa_n \beta_0^{n-1}\frac{\alpha_s^n(-s)}{\pi^n}$

Theoretical Description of R_τ II

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW}\left(1 + \delta_{\text{pert}} + \delta^{V/A}_{\text{mass}} + \delta^{V/A}_{\text{non-pert}}\right)$$

- power corrections

OPE: $\delta^{V/A}_{\text{mass}} \sim \frac{m_q^2}{m_\tau^2}$

$$\delta^{V/A}_{\text{non-pert}} \sim \underbrace{C_4^{V/A}\frac{\langle O\rangle^{D=4}}{m_\tau^4}}_{\langle\alpha_s GG\rangle,\ \ldots} + \underbrace{C_6^{V/A}\frac{\langle O\rangle^{D=6}}{m_\tau^6}}_{\delta^6_{V/A}} + \underbrace{C_8^{V/A}\frac{\langle O\rangle^{D=8}}{m_\tau^8}}_{\delta^8_{V/A}}$$

⇒ need several observables with different s dependence

The Definition of Moments of R_τ

$$R^{kl}_{\tau,V/A} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}$$

Cauchy Integral theorem

$$\oint \frac{ds}{s^n} = 0, \text{ for } n \neq 1$$

$$\delta^{D,kl}_{V/A} = 8\pi^2 \sum_{\dim O = D} \frac{C_D^{V/A} \langle O \rangle}{m_\tau^D} \begin{pmatrix} 1 & 0 & -3 & -2 \\ 1 & 1 & -3 & -5 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

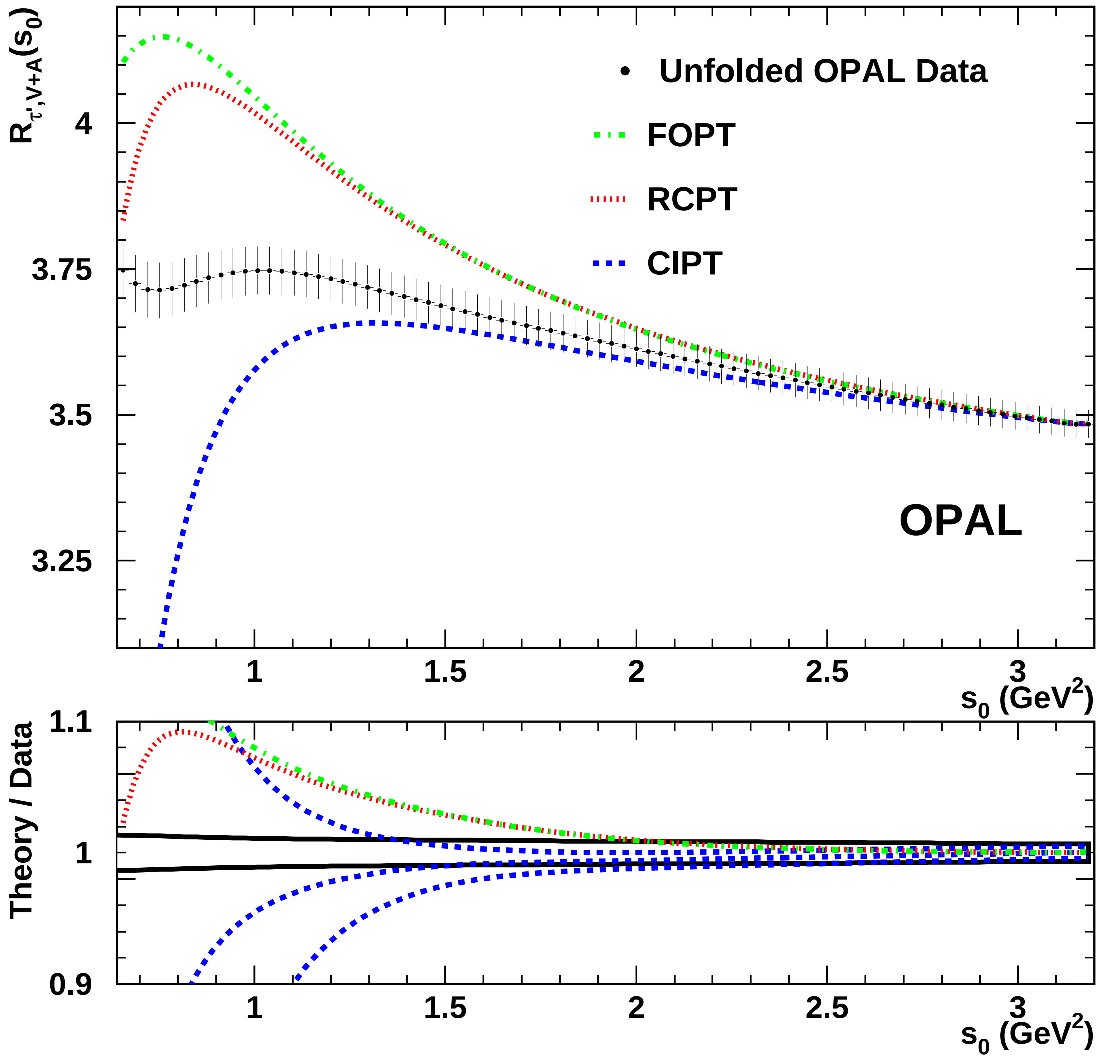
$D=2$	$D=4$	$D=6$	$D=8$	kl
1	0	−3	−2	00
1	1	−3	−5	10
0	−1	−1	3	11
0	0	1	1	12
0	0	0	−1	13

Comparison of different fits to the τ data (CIPT)

	V	A	V and A
$\alpha_s(m_\tau^2)$	0.341 ± 0.017	0.357 ± 0.019	0.347 ± 0.012
$\frac{\langle \alpha_s GG \rangle}{\mathrm{GeV}^4}$	0.002 ± 0.010	-0.011 ± 0.020	0.001 ± 0.008
δ_V^6	0.0259 ± 0.0041	—	0.0256 ± 0.0034
δ_V^8	-0.0078 ± 0.0018	—	-0.0080 ± 0.0013
δ_A^6	—	-0.0246 ± 0.0086	-0.0197 ± 0.0033
δ_A^8	—	0.0067 ± 0.0050	0.0041 ± 0.0019
$\chi^2/\mathrm{d.o.f.}$	$0.07/1$	$0.06/1$	$0.63/4$

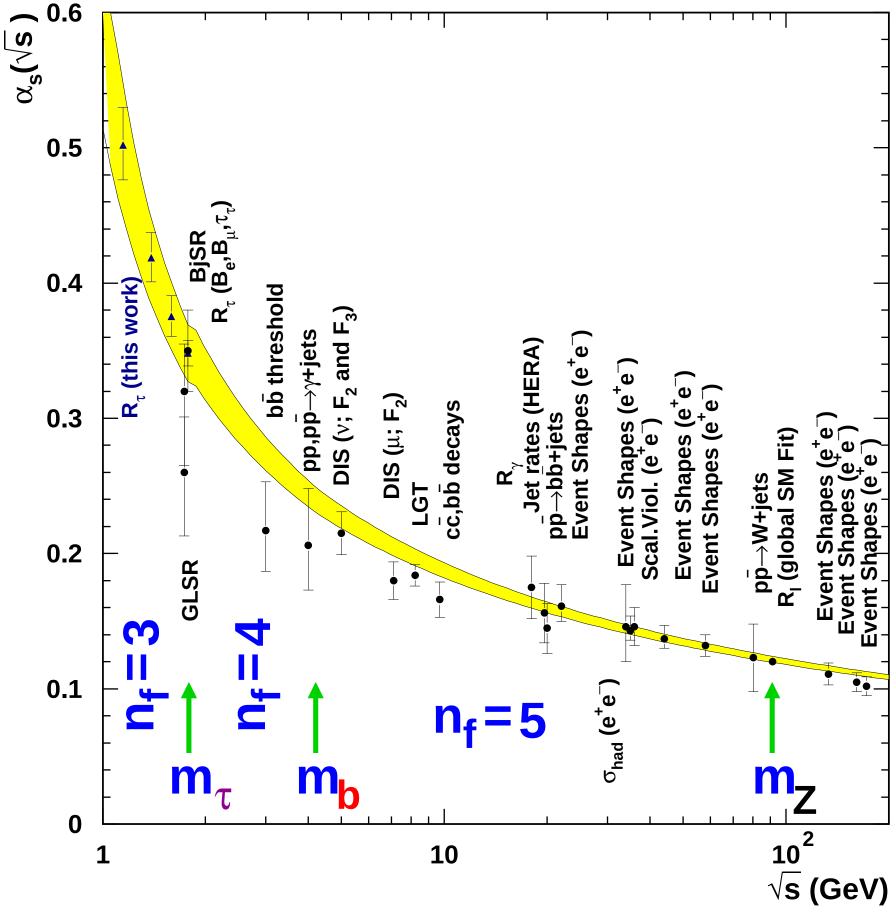
V+A		Stat.	Br.	Syst.	Theo.	$\chi^2/\mathrm{d.o.f.}$
$\alpha_s(m_\tau^2)$	0.348	± 0.002	± 0.009	± 0.002	± 0.019	$0.16/1$
$\frac{\langle \alpha_s GG \rangle}{\mathrm{GeV}^4}$	-0.003	± 0.007	± 0.007	± 0.006	± 0.005	
δ_{V+A}^6	0.0012	± 0.0034	± 0.0033	± 0.0029	± 0.0006	
δ_{V+A}^8	-0.0010	± 0.0024	± 0.0016	± 0.0015	± 0.0003	

Decay Ratio for a hypothetical τ'



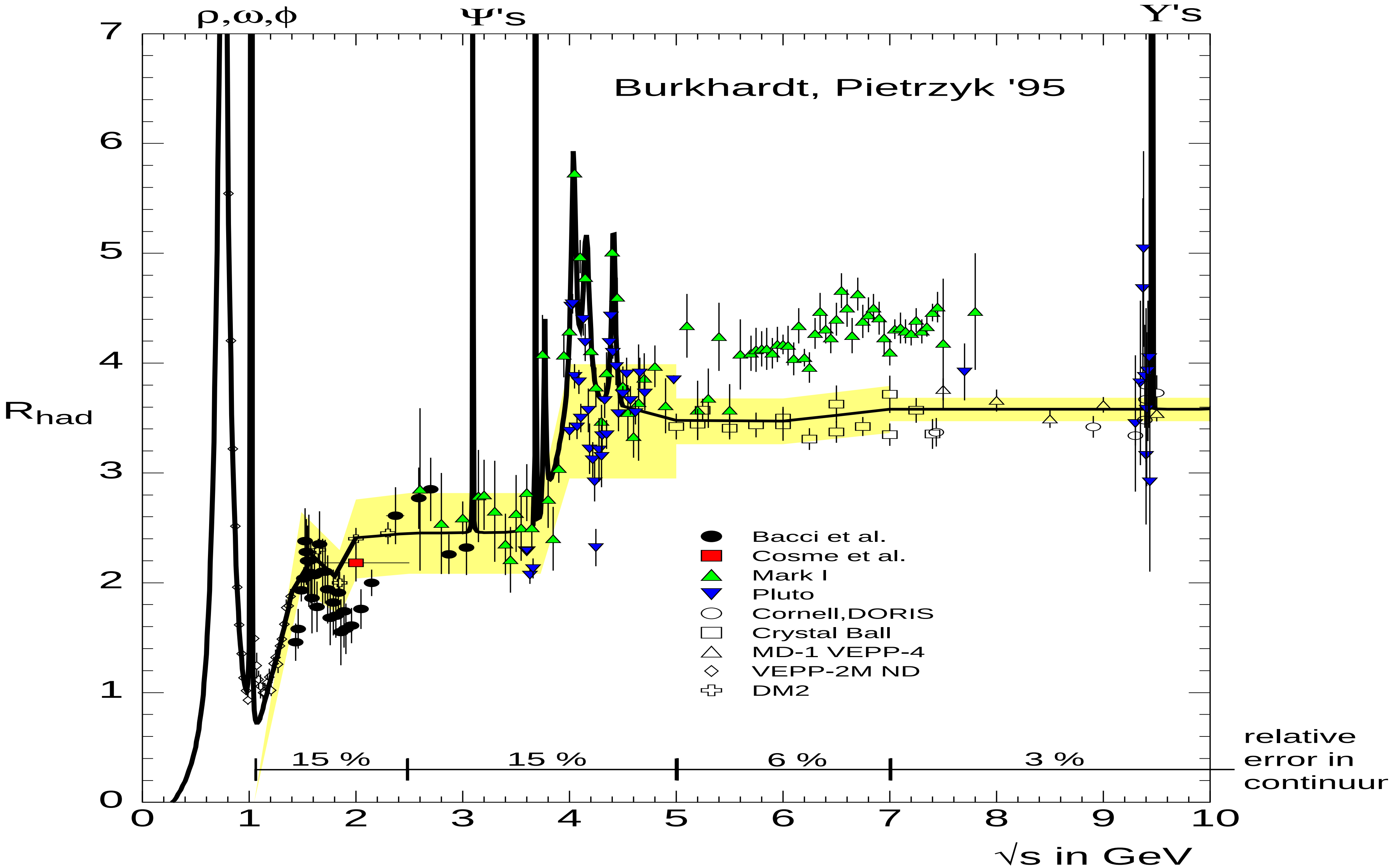
$$R_{\tau',V+A}(s_0 = m^2_{\tau'}) \sim \int_0^{s_0} \frac{ds}{s_0}\left(1 - \frac{s}{s_0}\right)^2 \left[\left(1 + \frac{2s}{s_0}\right)(v(s) + a(s)) + a^{(0)}(s)\right]$$

Running α_s



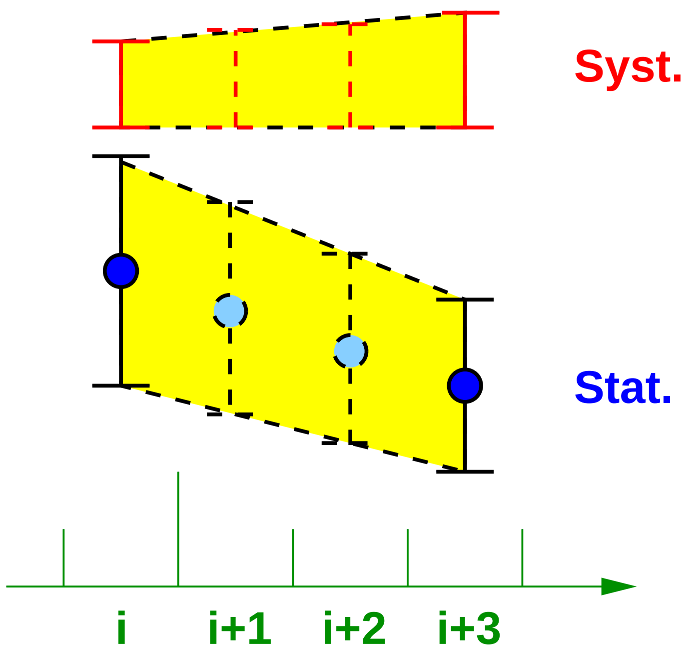
$\alpha_s(m_Z)$		exp.	theo.	evol.
CIPT:	0.1219	±0.0010	±0.0017	±0.0003
FOPT:	0.1191	±0.0008	±0.0013	±0.0003
RCPT:	0.1169	±0.0007	±0.0015	±0.0003

$R_{e^+e^-}$ and α_s



$$R_{e^+e^-}(s) = 12\pi\,\mathrm{Im}\Pi_\gamma(s) = N_C \sum_f Q_f^2 \left[1 + \frac{\alpha_s(s)}{\pi} + d_2 \frac{\alpha_s^2(s)}{\pi^2} + d_3 \frac{\alpha_s^3(s)}{\pi^3} \right]$$

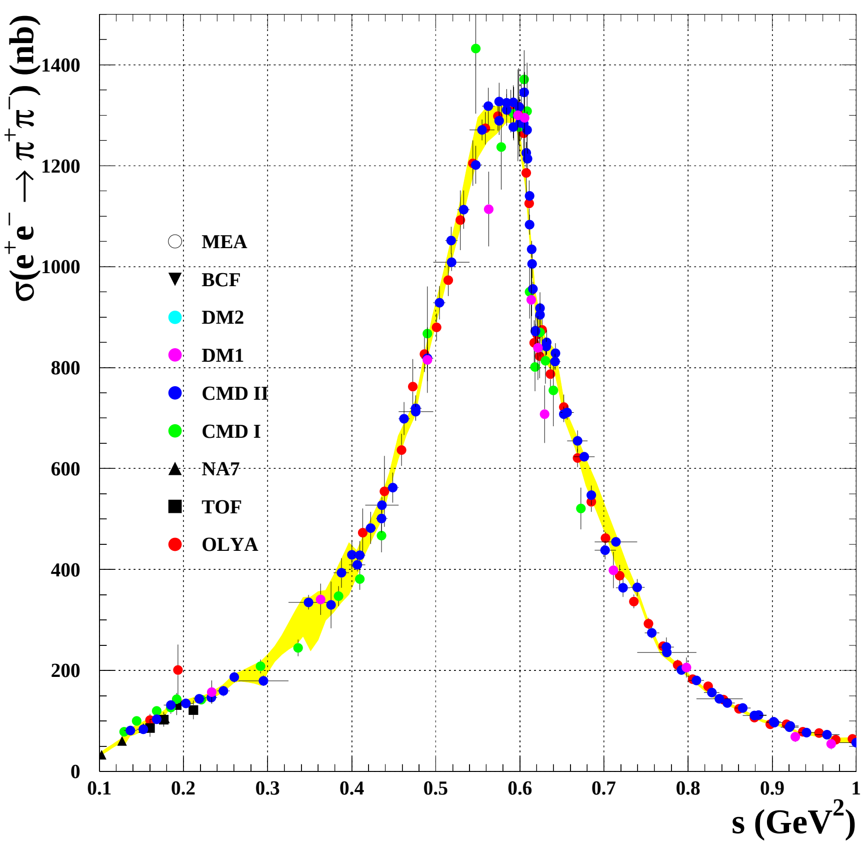
Averaging of the e^+e^- data

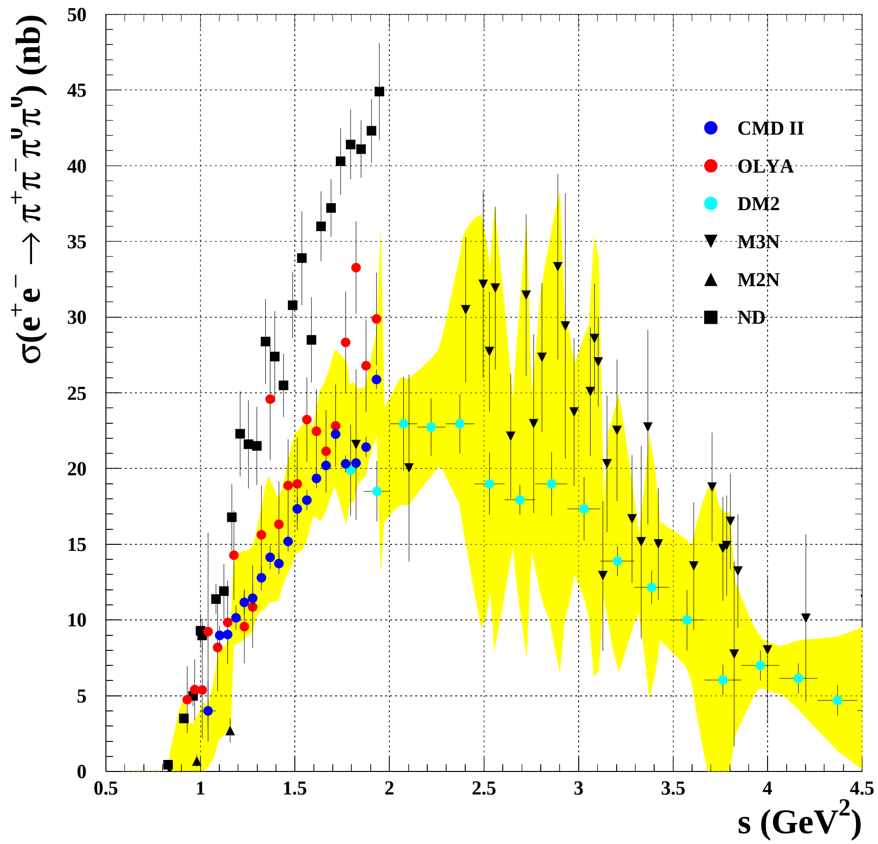


- individual experiments interpolated with trapezoidal rule
- data points:
  $d_k = c_k\, d_i + (1 - c_k)\, d_{i+3}$, $k = i+1, i+2$
- statistical errors:
  $\sigma_k = c_k\, \sigma_i + (1 - c_k)\, \sigma_{i+3}$
- larger than the Gauss errors:
  $\sigma'_k = c_k\, \sigma_i \oplus (1 - c_k)\, \sigma_{i+3}$
  Ratio: $r_k = \sigma_k / \sigma'_k$
- Correlation matrix:
  $V^{stat}_{kl} = r_k r_l \left(c_k c_l \sigma_i^2 + (1 - c_k)(1 - c_l)\sigma_{i+3}^2 \right)$

- systematic errors are also interpolated with trapezoidal rule
- correlation is $100\,\%$
- Total Error matrix: $V = V^{stat} + V^{sys}$
- Weighted average over all individual results gives final data points
- Errors scaled with $S = \sqrt{\chi^2/\chi^2_{68\,\%}}$ if $S > 1$

Exclusive modes I: 2π, $2\pi 2\pi^0$

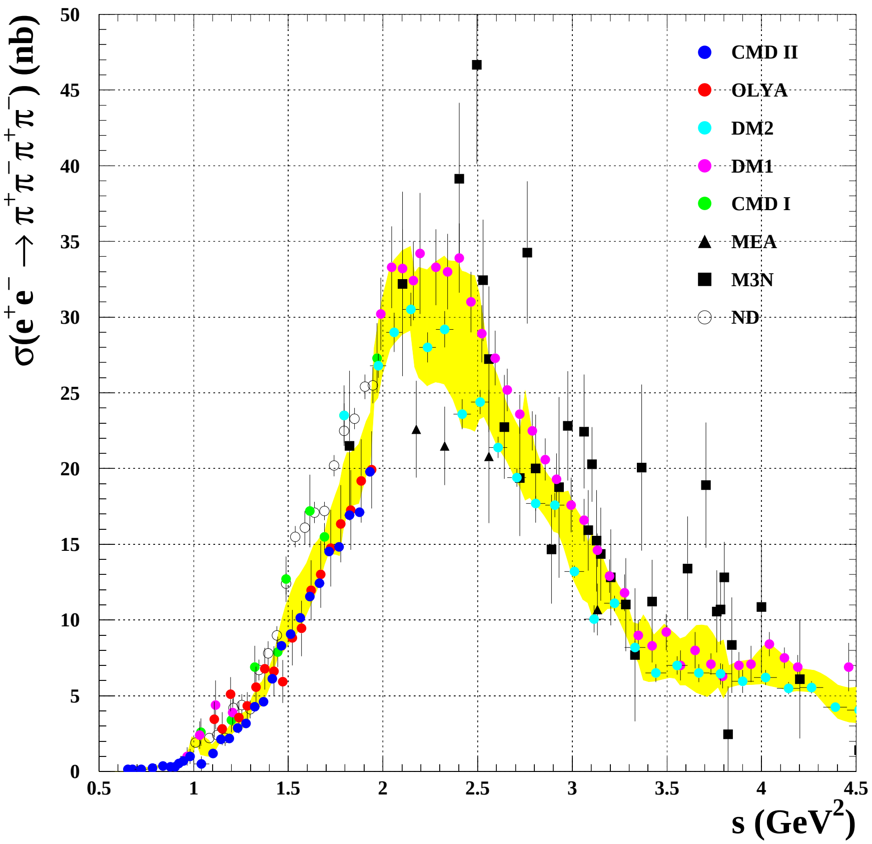


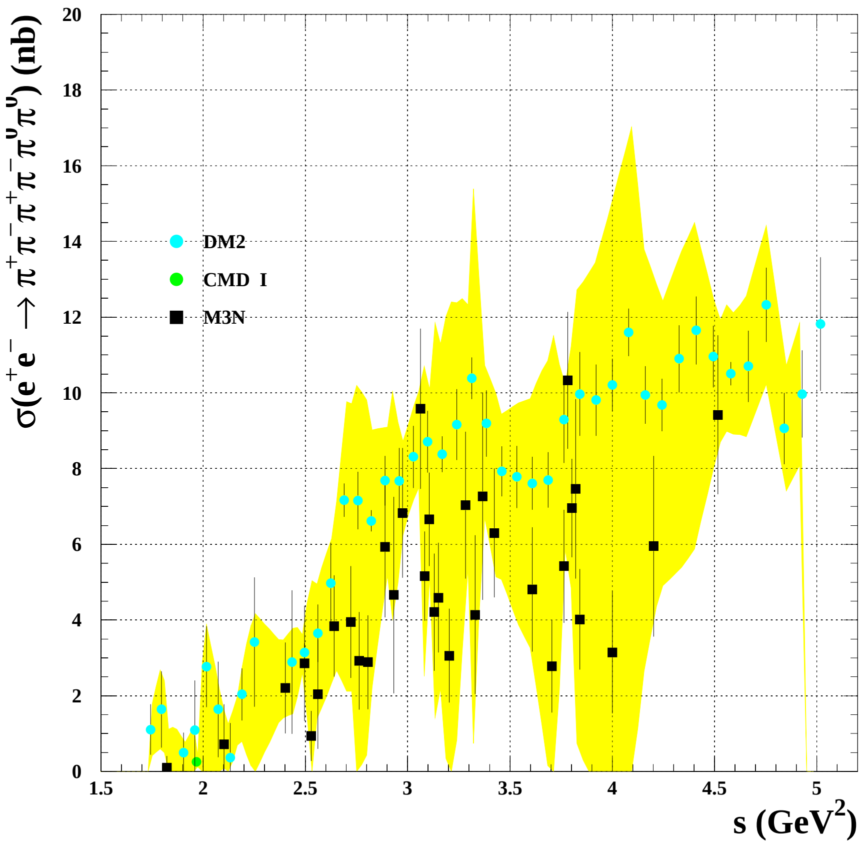


$\sigma(e^+e^- \to \pi^+\pi^-)(s)$

$\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0)(s)$

Exclusive modes II: 4π, $4\pi 2\pi^0$

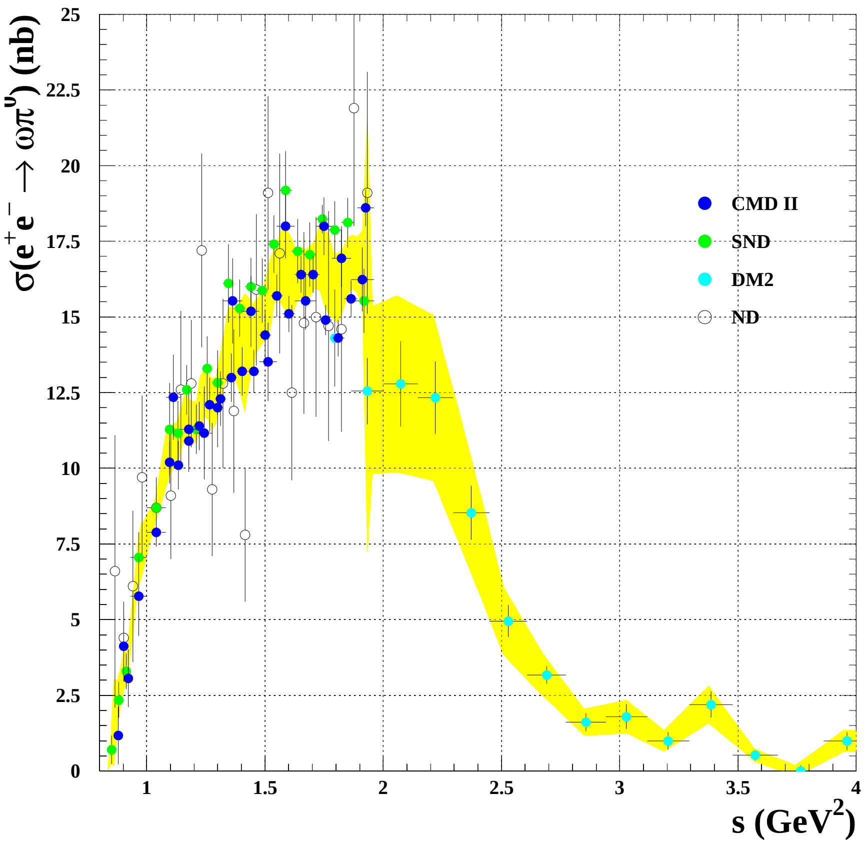


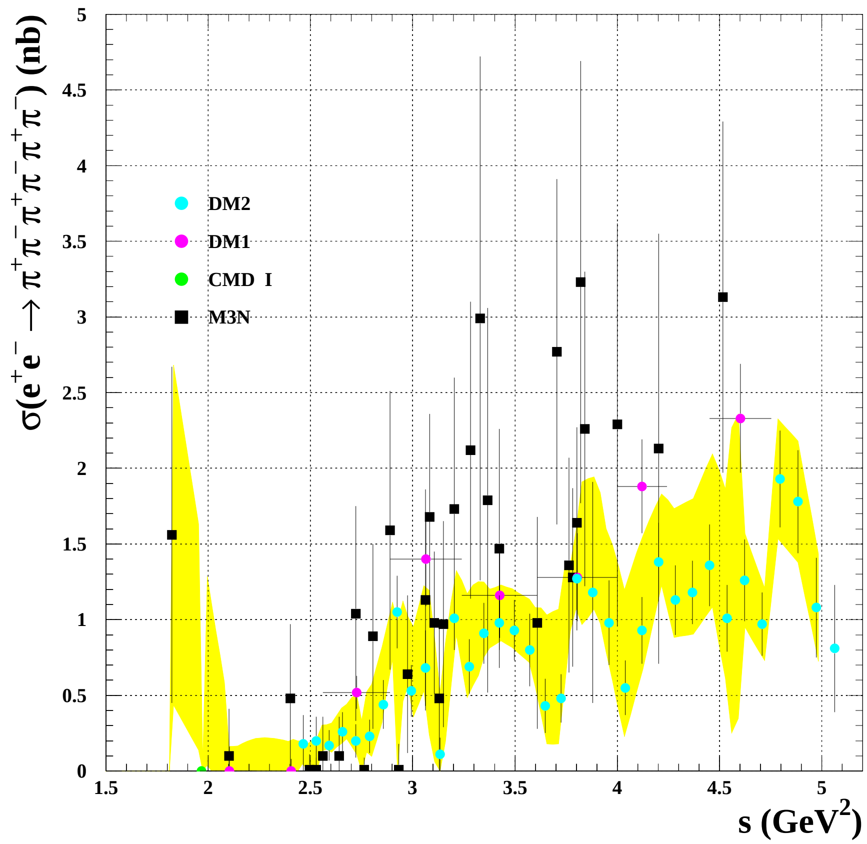


$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)(s)$

$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0)(s)$

Exclusive modes III: $\omega\pi^0$, 6π

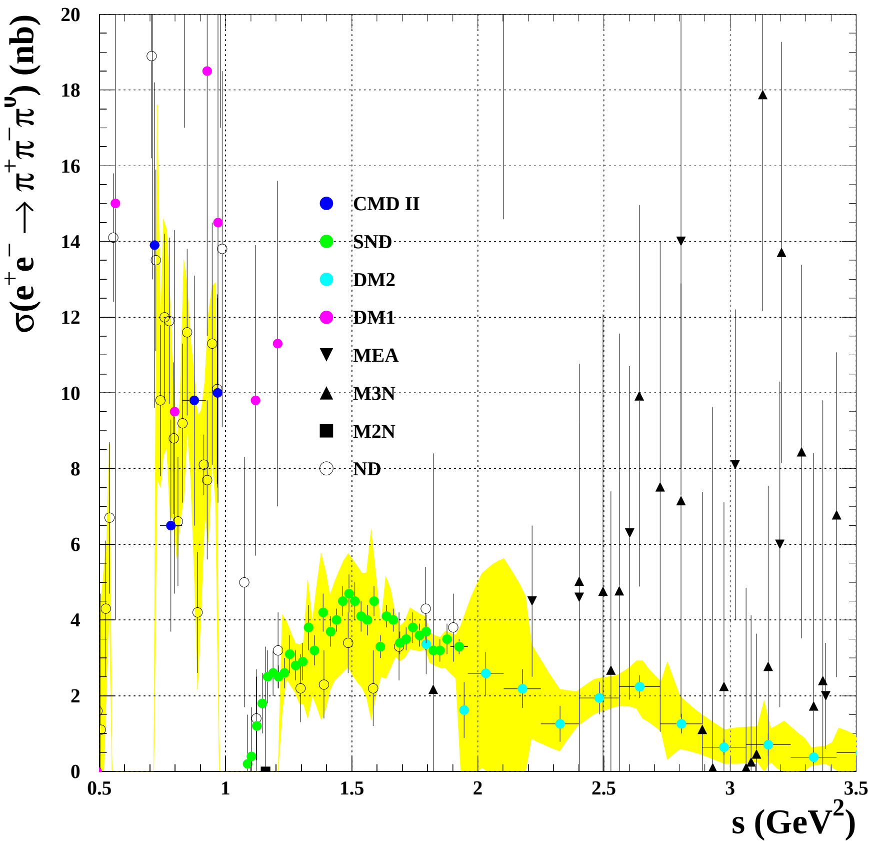


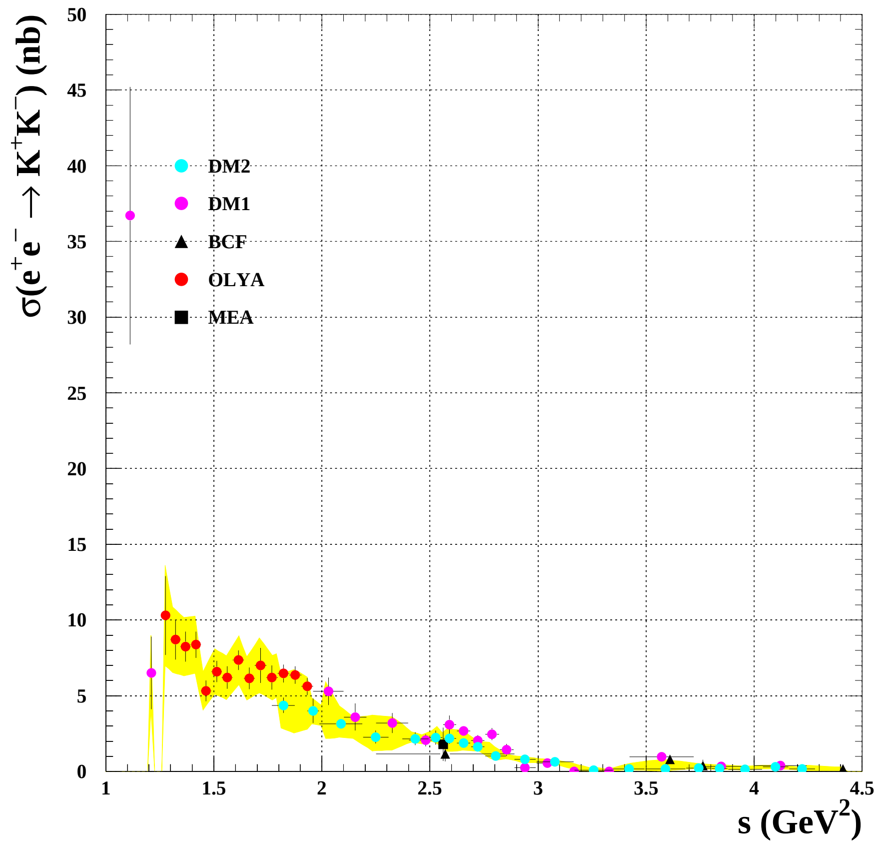


$\sigma(e^+e^- \to \omega\pi^0)(s)$

$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-)(s)$

Exclusive modes IV: $2\pi\pi^0$, 2K

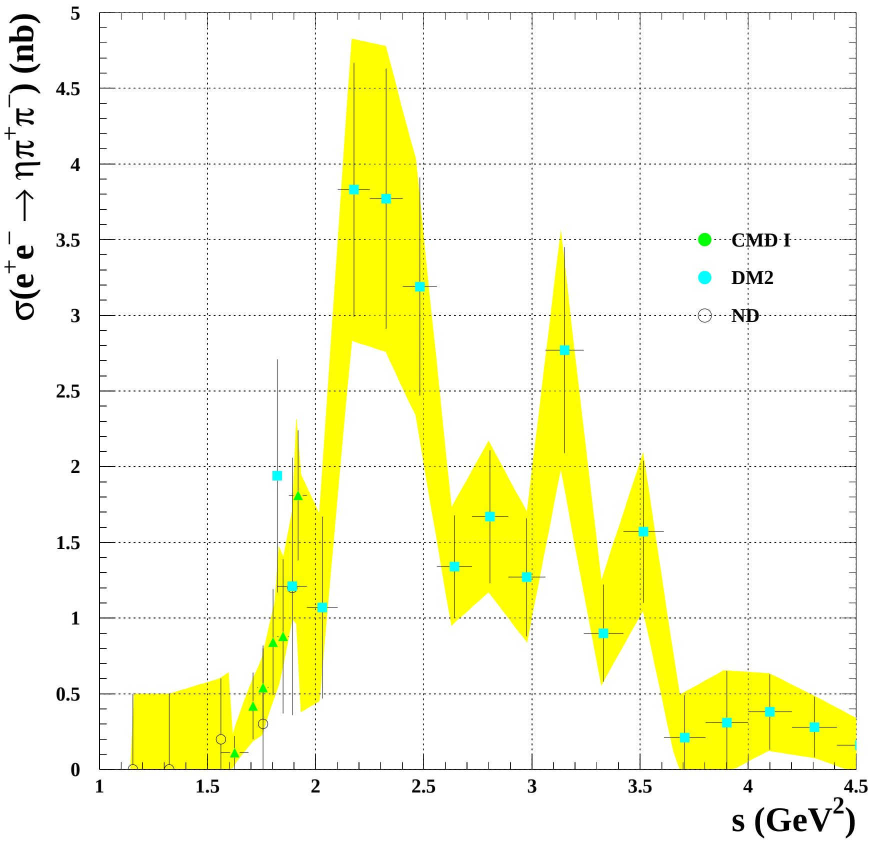




$\sigma(e^+e^- \to \pi^+\pi^-\pi^0)(s)$

$\sigma(e^+e^- \to K^+K^-)(s)$

Exclusive modes V: $\eta 2\pi$

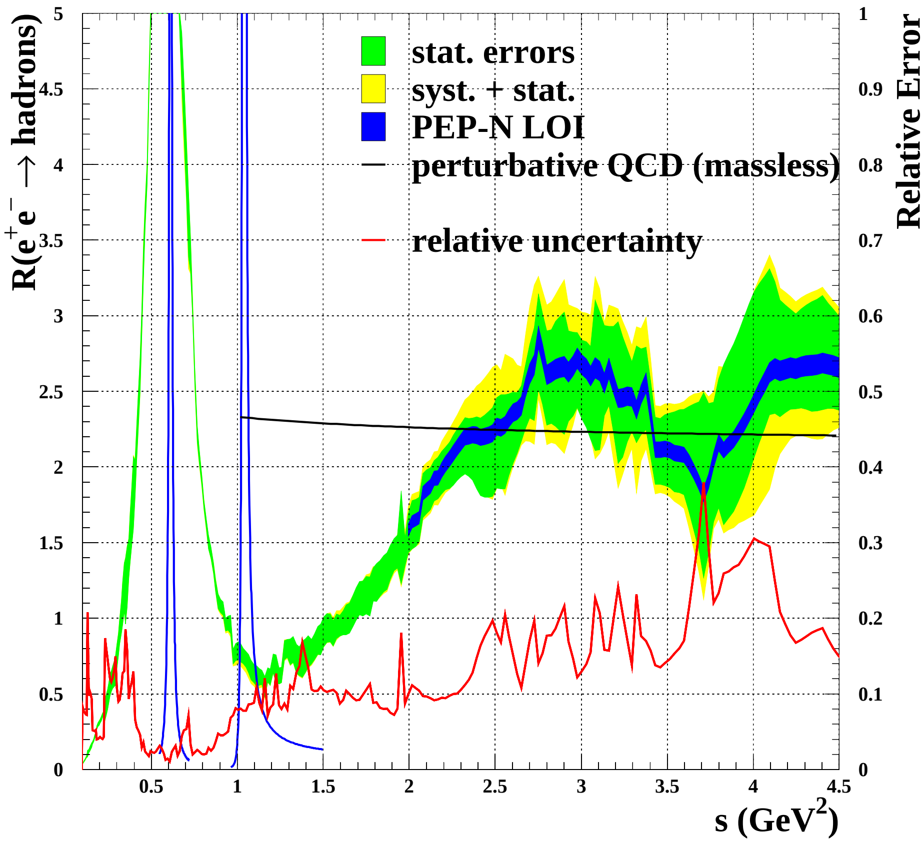


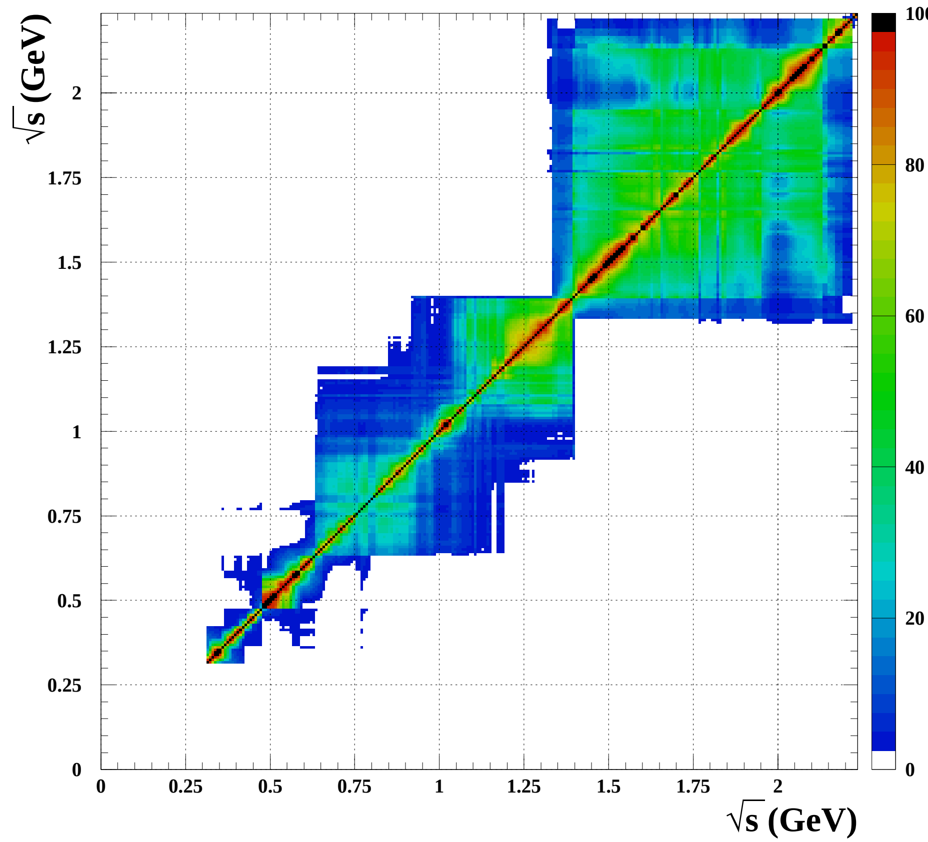
$\sigma(e^+e^- \to \eta\pi^+\pi^-)(s)$

Other exclusive modes without picture

- $4\pi\pi^0$
- $3\pi 2\pi^0$
- $2\pi 3\pi^0$
- $K_S^0 K_L^0$
- $2K\pi^0$
- $2K2\pi$
- $K_S^0 X$

$R_{e^+e^-}$ from exclusive modes preliminary





$R_{e^+e^-}(s)$

Correlations in %

The Definition of Moments of $R_{e^+e^-}$

$$R^{kl}_{e^+e^-}(s_0) = \int_0^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^l R_{e^+e^-}(s)$$

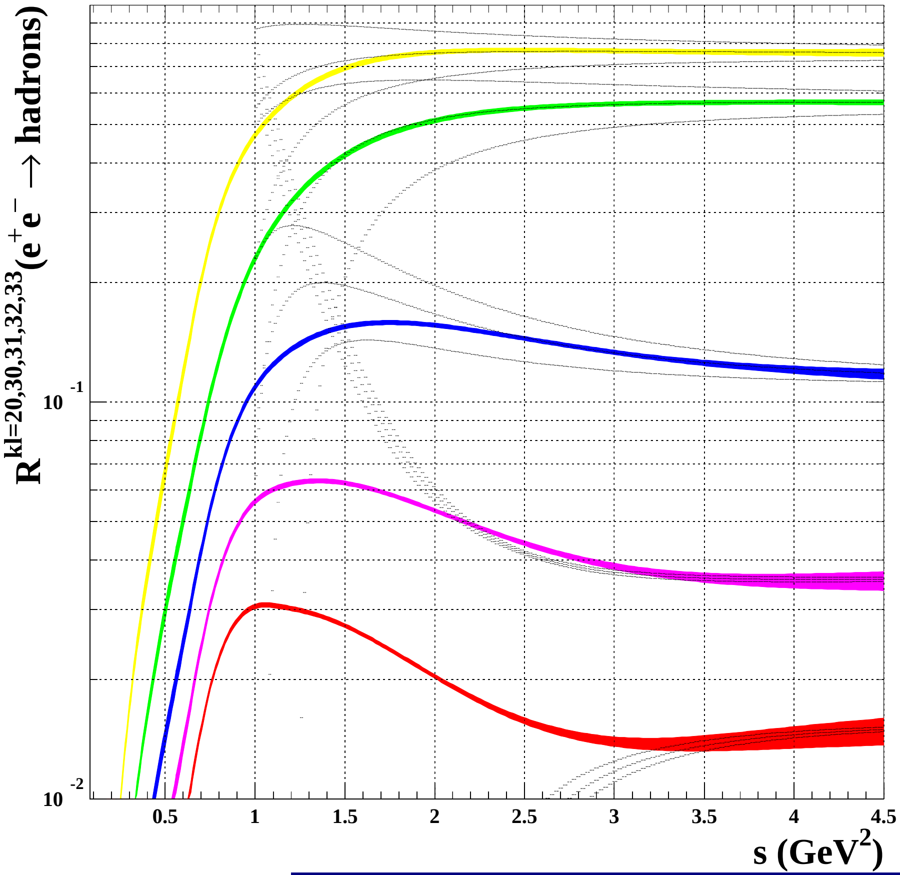
$$= 6\pi i \oint_{|s|=s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^l \Pi_\gamma(s)$$

$$\delta_\gamma^{D,kl} = 12\pi^2 \sum_{\dim O = D} \frac{C_D^\gamma \langle O \rangle}{s_0^{D/2}} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \\ 0 & -1 & -3 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

D = 2	D = 4	D = 6	D = 8	kl
1	2	1	0	20
1	3	3	1	30
0	−1	−3	−3	31
0	0	1	3	32
0	0	0	−1	33

inspired by M. Davier and A. Höcker, Phys. Lett. **B419** (1998) 419.

$R^{kl=20,30,31,32,33}_{e^+e^-}$ up to 4.5 GeV^2 I preliminary



- colored curves denote data and experimental errors
- black curves show extrapolated fit results from a fit at $s_0 = 4\,GeV^2$ with theoretical and experimental errors
- theoretical errors (strange quark mass) dominant

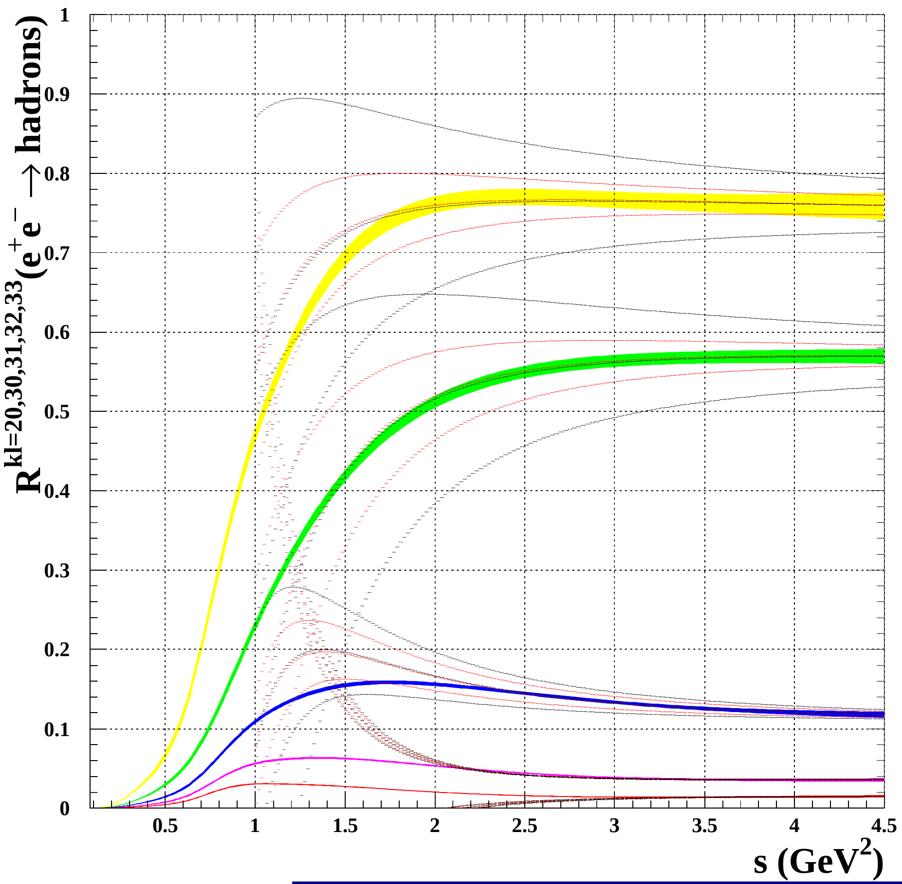
$$\alpha_s(m_Z) = 0.118 \pm 0.003 \pm 0.005$$

$$\langle \alpha_s GG \rangle = (0.028 \pm 0.006 \pm 0.027)\,GeV^4$$

$$\langle O \rangle^{D=6} = -(0.0042 \pm 0.0008 \pm 0.0001)\,GeV^6$$

$$\langle O \rangle^{D=8} = (0.0044 \pm 0.0005 \pm 0.0006)\,GeV^8$$

$R^{kl=20,30,31,32,33}_{e^+e^-}$ up to 4.5 GeV^2 II preliminary



- black curves show extrapolated fit results from a fit at $s_0 = 4\,\text{GeV}^2$ with $\langle \alpha_s GG \rangle$ as free parameter
- red curves show extrapolated fit results from a fit at $s_0 = 4\,\text{GeV}^2$ with m_s as free parameter

$$\alpha_s(m_Z) = 0.117 \pm 0.004 \pm 0.002$$

$$m_s = (0.220 \pm 0.036 \pm 0.056)\,\text{GeV}$$

$$\langle O \rangle^{D=6} = -(0.0041 \pm 0.0007 \pm 0.0001)\,\text{GeV}^6$$

$$\langle O \rangle^{D=8} = (0.0043 \pm 0.0005 \pm 0.0002)\,\text{GeV}^8$$

Conclusions

- OPE and QCD fits work reliable at low s
- $s_0^{min} \approx 1.5\,\mathrm{GeV}^2$ for non-strange τ decays
- $s_0^{min} \approx 1.5 - 3\,\mathrm{GeV}^2$ for e^+e^- annihilation (choice of kl)
- experimental error from low s e^+e^- data on $\alpha_s(m_Z) \approx 3\,\%$
- expected improvement of experimental errors at PEP-N:

$$\begin{aligned} \Delta\alpha_s &\longrightarrow 0.93\Delta\alpha_s \\ \Delta\langle\alpha_s\,GG\rangle &\longrightarrow 0.72\Delta\langle\alpha_s\,GG\rangle \\ \Delta\langle O\rangle^{D=6} &\longrightarrow 0.70\Delta\langle O\rangle^{D=6} \\ \Delta\langle O\rangle^{D=8} &\longrightarrow 0.56\Delta\langle O\rangle^{D=8} \end{aligned}$$

- sensitivity to non-perturbative QCD parameters could be used to extract m_s

⇒ fit with m_s instead of $\langle\alpha_s\,GG\rangle$ as free parameter and
$\langle\alpha_s\,GG\rangle = (0.02 \pm 0.01)\,\mathrm{GeV}^4$
gives $m_s = (220 \pm 36(26) \pm 59)\,\mathrm{MeV}$