Measurement

of the

Non-Strange Spectral Functions

in

Hadronic τ Decays

with

OPAL

QCD99 7-13th July 1999, Montpellier Sven Menke, SLAC

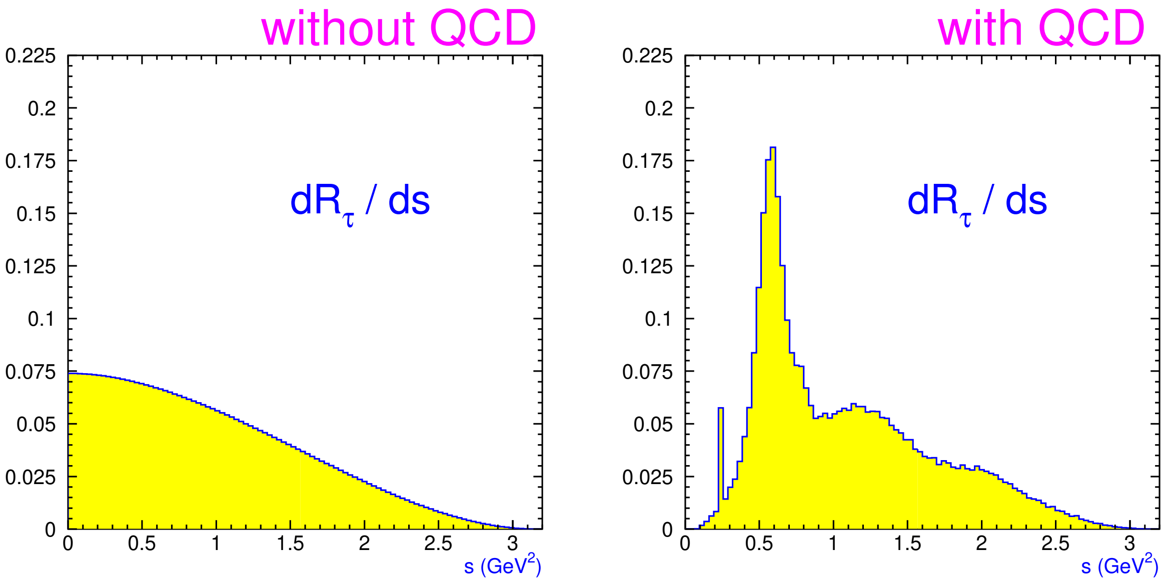
- Introduction
- Event Selection / Reconstruction
- Spectral Functions
- Applications
  - $\alpha_s(m_\tau^2)$
  - Running α_s
  - Non-perturbative QCD
  - Sum Rules

Eur. Phys. J. **C7** (1999) 571.

The Hadronic Decay Ratio of the τ

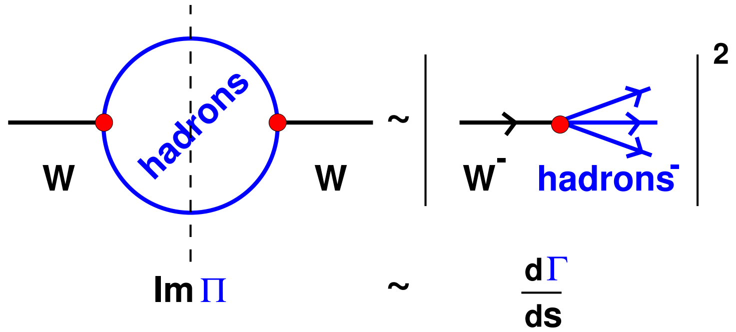
$$R_\tau = \frac{\Gamma\left(\tau \to \nu_\tau \text{ hadrons}\right)}{\Gamma\left(\tau \to \nu_\tau \, e \, \nu_e\right)}$$

- Tree: $R_\tau = N_C \left(|V_{ud}|^2 + |V_{us}|^2\right)$
  $= 3$
- Exp.: $R_\tau = 3.65$



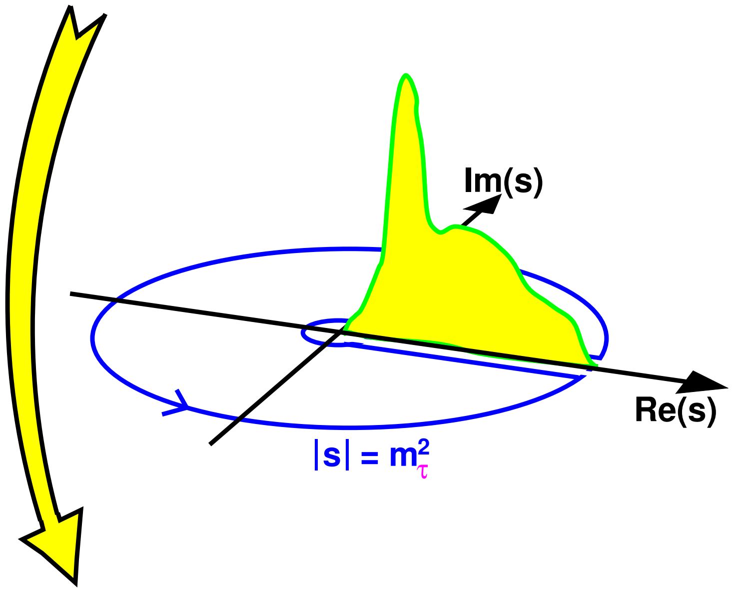
R_τ and the Spectral Functions

Optical theorem



$$R_\tau = 12\pi \int_0^{m_\tau^2} ds \; \mathrm{poly}(s) \; \mathrm{Im}\Pi(s)$$

Cauchy's theorem



$$R_\tau = 6\pi i \oint_{|s|=m_\tau^2} ds \; \mathrm{poly}(s) \; \Pi(s)$$

Event Selection

- τ pair selection
  - 2 narrow jets

~~Multi-hadron~~ up to 6 tracks per event

  - back-to-back topology

~~Bhabhas~~ $\mathrm{E} < 0.8 \times 2\mathrm{E}_{\mathrm{beam}}$

~~μ pairs~~ $\mathrm{p} < 0.6 \times 2\mathrm{E}_{\mathrm{beam}}$

or > 2 tracks or no μ-ID

  - $|\cos\theta| < 0.95$

297988 τ candidates

- τ decay mode identification
  - Maximum Likelihood selection
  - 6 one-prong modes: e, μ, h, hπ^0, h2π^0, h3π^0
  - 8 three-prong modes: 5 one-prong modes + conv., 3h(ex K^0), K^0h$\rightarrow$3h, 3h$\geq 1\pi^0$
  - π/K separation by dE/dx

- Selected modes
  - $\pi\pi^0, \pi 2\pi^0, \pi 3\pi^0, 3\pi(\mathrm{ex}\ K^0), 3\pi\pi^0, 3\pi 2\pi^0$

65899 τ candidates

Contributing Channels

Vector:	$B(\tau \to h_V) =$	$(30.6 + 0.8)\,\%$
	$B(\tau \to \pi\pi^0)$	$25.2\,\%$
$+$	$B(\tau \to 3\pi\pi^0(\text{excl. } \omega\pi\pi^0))$	$4.3\,\% - 0.01\,\%$
$+$	$B(\tau \to \pi 3\pi^0)$	$1.1\,\%$
$+$	$B(\tau \to 3\pi 3\pi^0)$	$0.1\,\%$
$+$	$B(\tau \to 5\pi\pi^0)$	$0.02\,\%$
$+(0.5 \pm 0.5)$	$B(\tau \to 2K\pi)$	$0.17\,\%$
$+$	$B(\tau \to 2K)$	$0.16\,\%$
$+$	$B(\tau \to \omega\pi(\text{excl. } 3\pi\pi 0))$	$0.2\,\%$
$+$	$B(\tau \to \eta\pi\pi^0)$	$0.17\,\%$
Axial Vector:	$B(\tau \to h_A) =$	$(30.4 + 0.3)\,\%$
	$B(\tau \to \pi)$	$11.3\,\%$
$+$	$B(\tau \to 3\pi(\text{excl. } \omega\pi))$	$9.26\,\% - 0.04\,\%$
$+$	$B(\tau \to \pi 2\pi^0)$	$9.27\,\%$
$+$	$B(\tau \to 3\pi 2\pi^0(\text{excl. } \eta\pi\pi^0))$	$0.50\,\% - 0.04\,\%$
$+$	$B(\tau \to \pi 4\pi^0)$	$0.12\,\%$
$+(0.5 \pm 0.5)$	$B(\tau \to 2K\pi)$	$0.17\,\%$
$+$	$B(\tau \to \omega\pi\pi^0(\text{excl. } 3\pi 2\pi^0))$	$0.05\,\%$
$+$	$B(\tau \to 5\pi)$	$0.08\,\%$

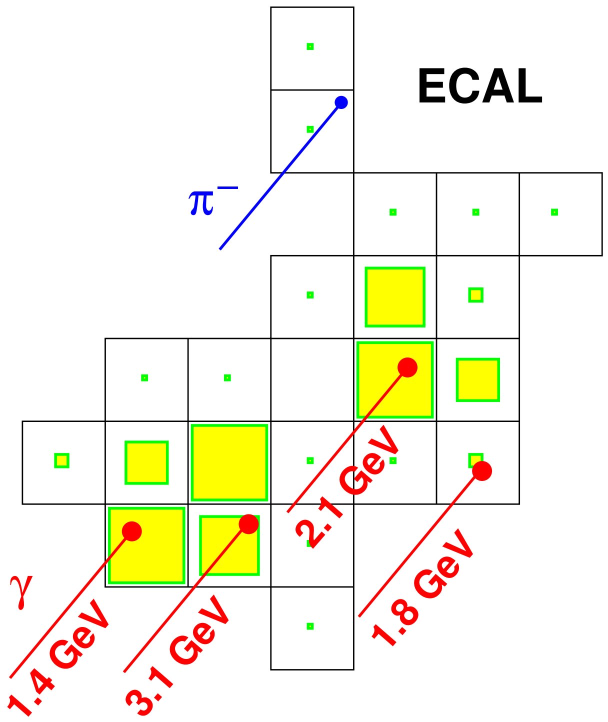
PDG: $R_\tau^V = 1.764 \pm 0.016, \quad R_\tau^A = 1.720 \pm 0.017$

Photon Reconstruction

Decay Modes

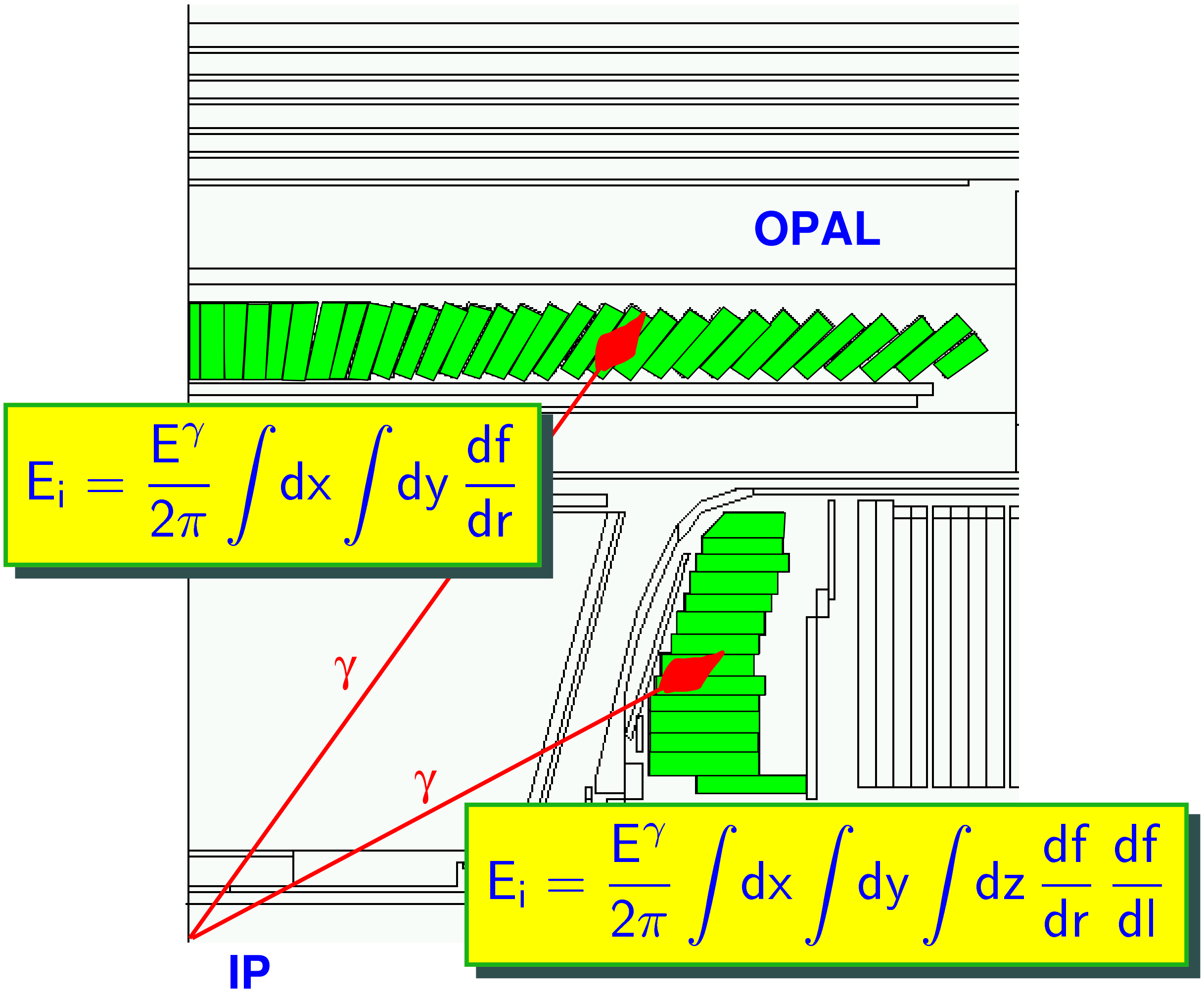
$$
\begin{aligned}
\tau^- \to\ & \pi^- \\
& \pi^-\ \pi^0 \\
& \qquad \to 2\gamma \\
& \pi^-\ 2\pi^0 \\
& \qquad \to 4\gamma \\
& \pi^-\ 3\pi^0 \\
& \qquad \to 6\gamma
\end{aligned}
$$

$$
\begin{aligned}
\tau^- \to\ & 2\pi^-\pi^+ \\
& 2\pi^-\pi^+\ \pi^0 \\
& \qquad\qquad \to 2\gamma \\
& 2\pi^-\pi^+\ 2\pi^0 \\
& \qquad\qquad \to 4\gamma
\end{aligned}
$$



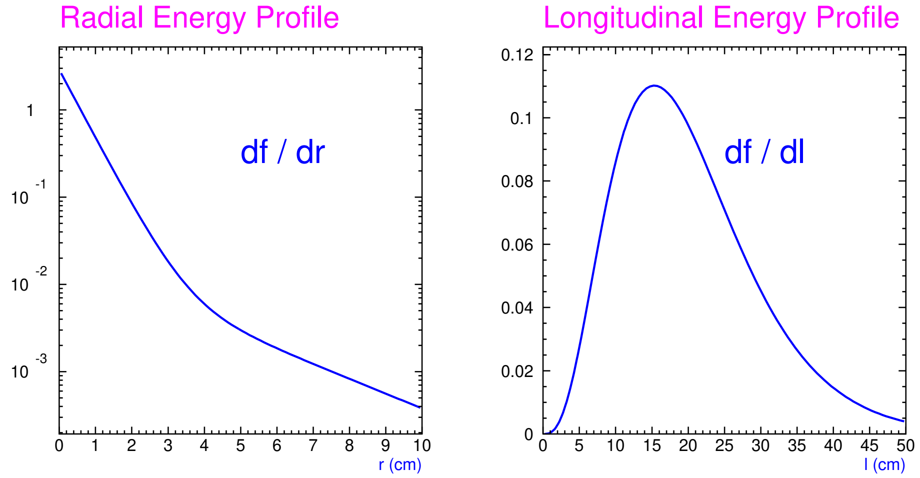
- Parameterization of electro-magnetic showers
- χ^2 fits of γ energies and directions to observed ECAL Block energies

Electro-magnetic Showers

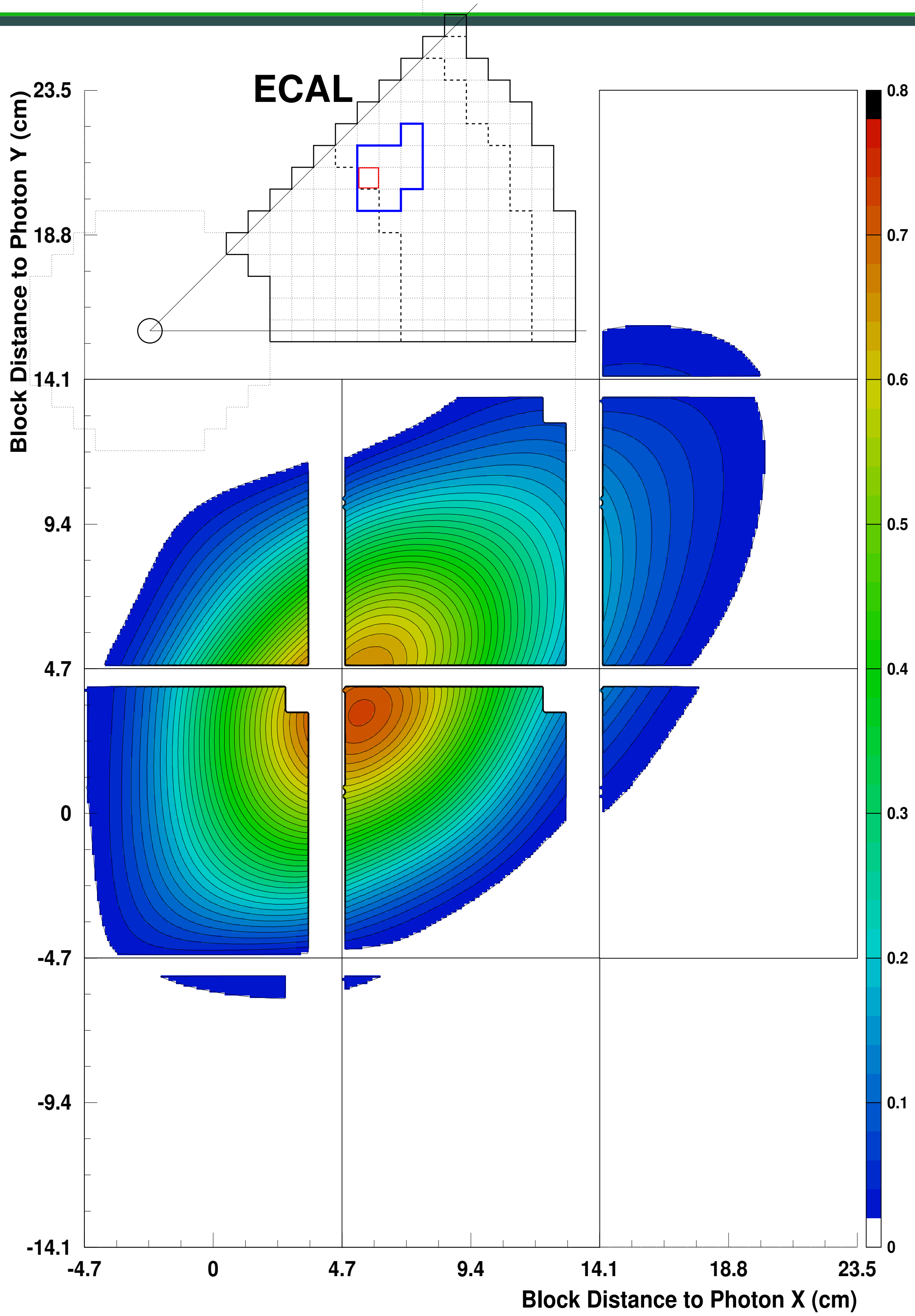


$$E_i = \frac{E^\gamma}{2\pi} \int dx \int dy \, \frac{df}{dr}$$

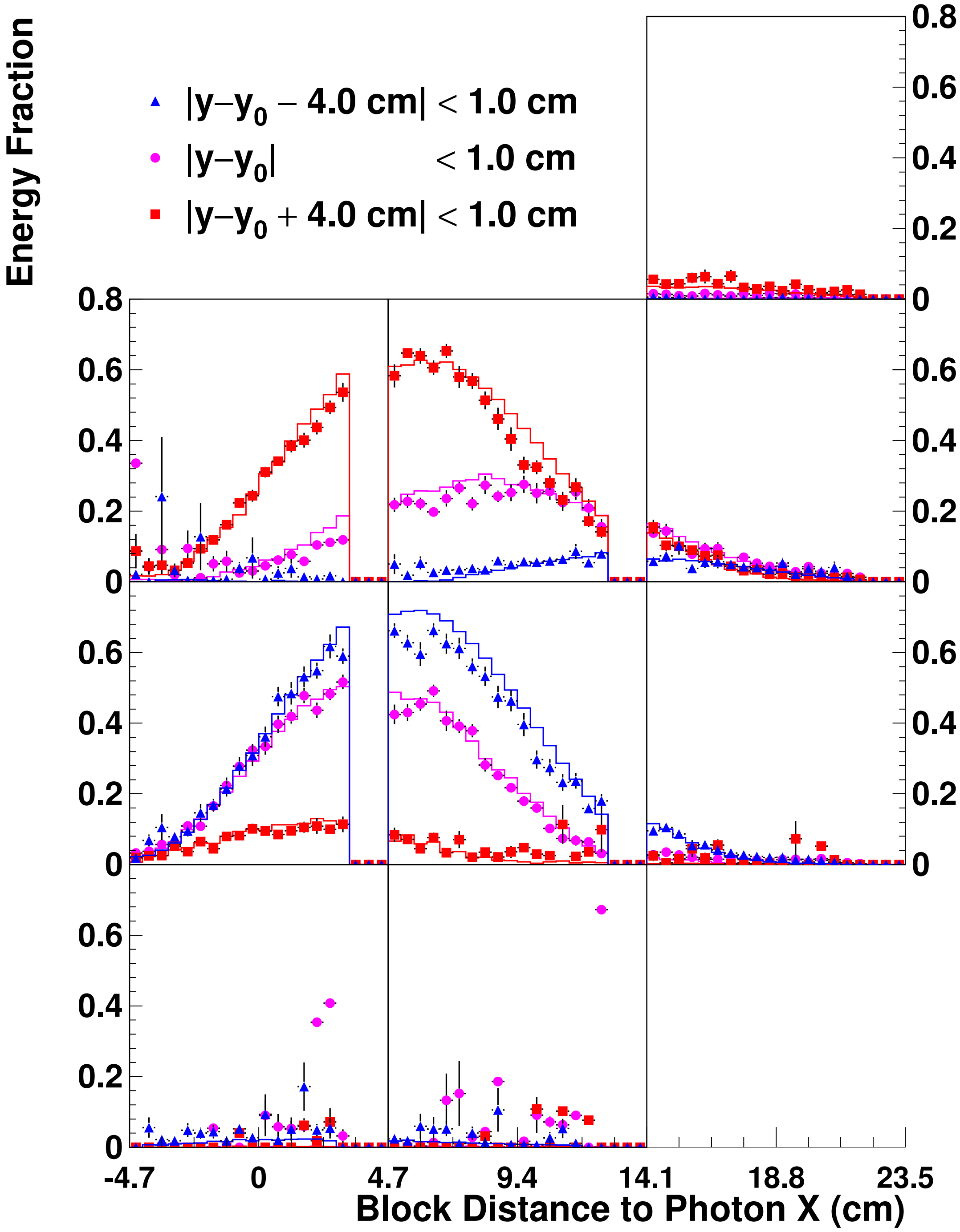
$$E_i = \frac{E^\gamma}{2\pi} \int dx \int dy \int dz \, \frac{df}{dr} \, \frac{df}{dl}$$



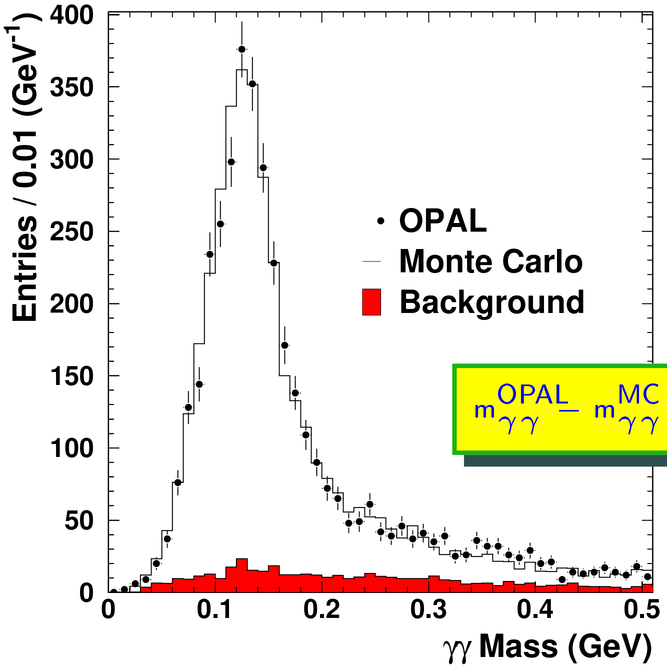
Shower Parameterization



Comparison with full Detector Simulation



π^0 *Reconstruction*



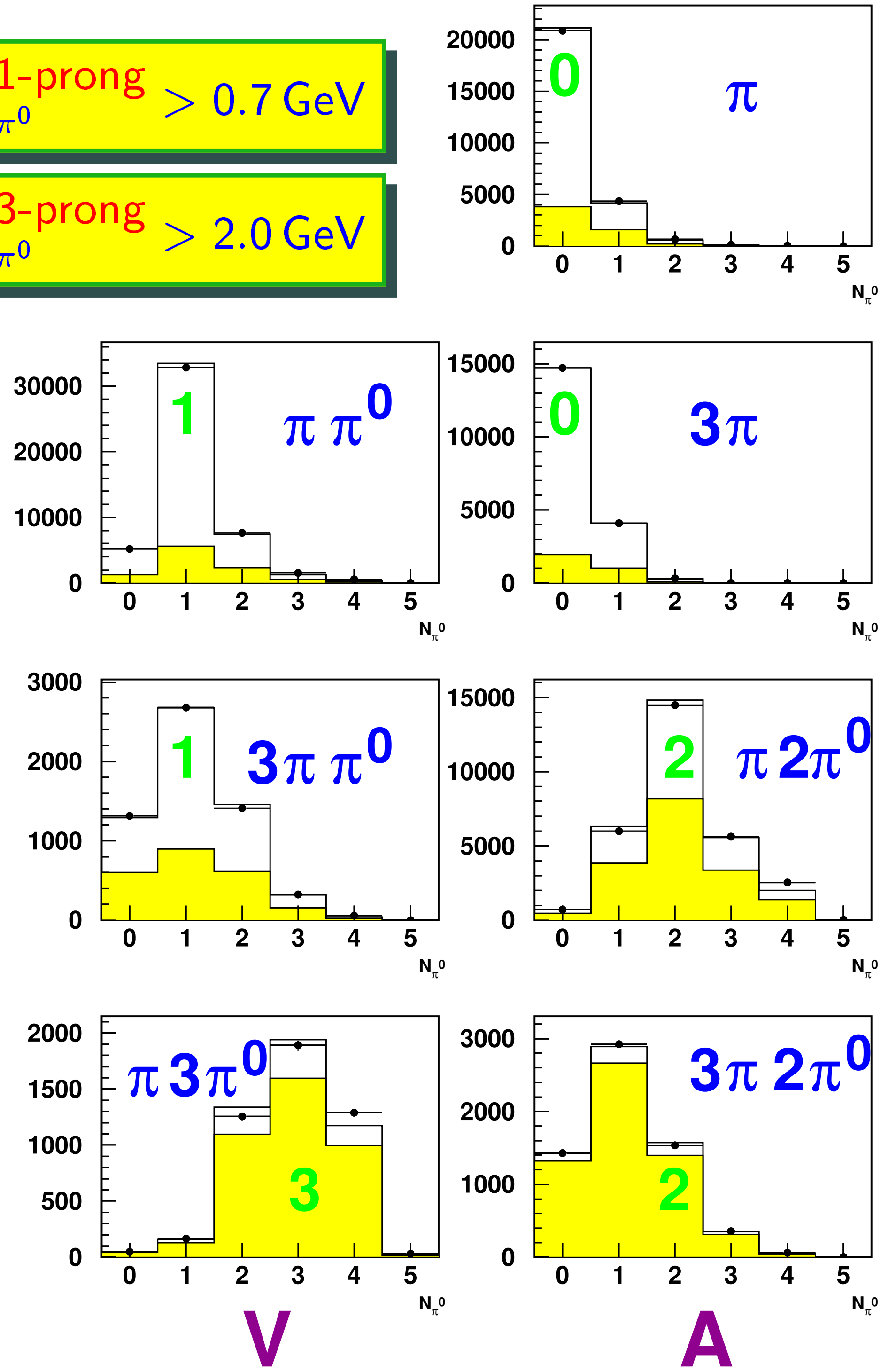
$m_{\gamma\gamma}^{OPAL} - m_{\gamma\gamma}^{MC} = (0.5 \pm 0.9)\,\mathrm{MeV}$

- find the smallest number of photons consistent with the observed ECAL energies
- find the largest number of π^0s from all possible $\gamma\gamma$ combinations and an average deviation of $|m_{\gamma\gamma} - m_{\pi^0}| < 1.5\sigma$
- constrain $m_{\gamma\gamma}$ to m_{π^0} $\rightarrow$ improve π^0 4 vector
- $E_{\pi^0} > 12\,\mathrm{GeV}$ $\rightarrow$ merged photons $\rightarrow$ 'single photon' π^0s

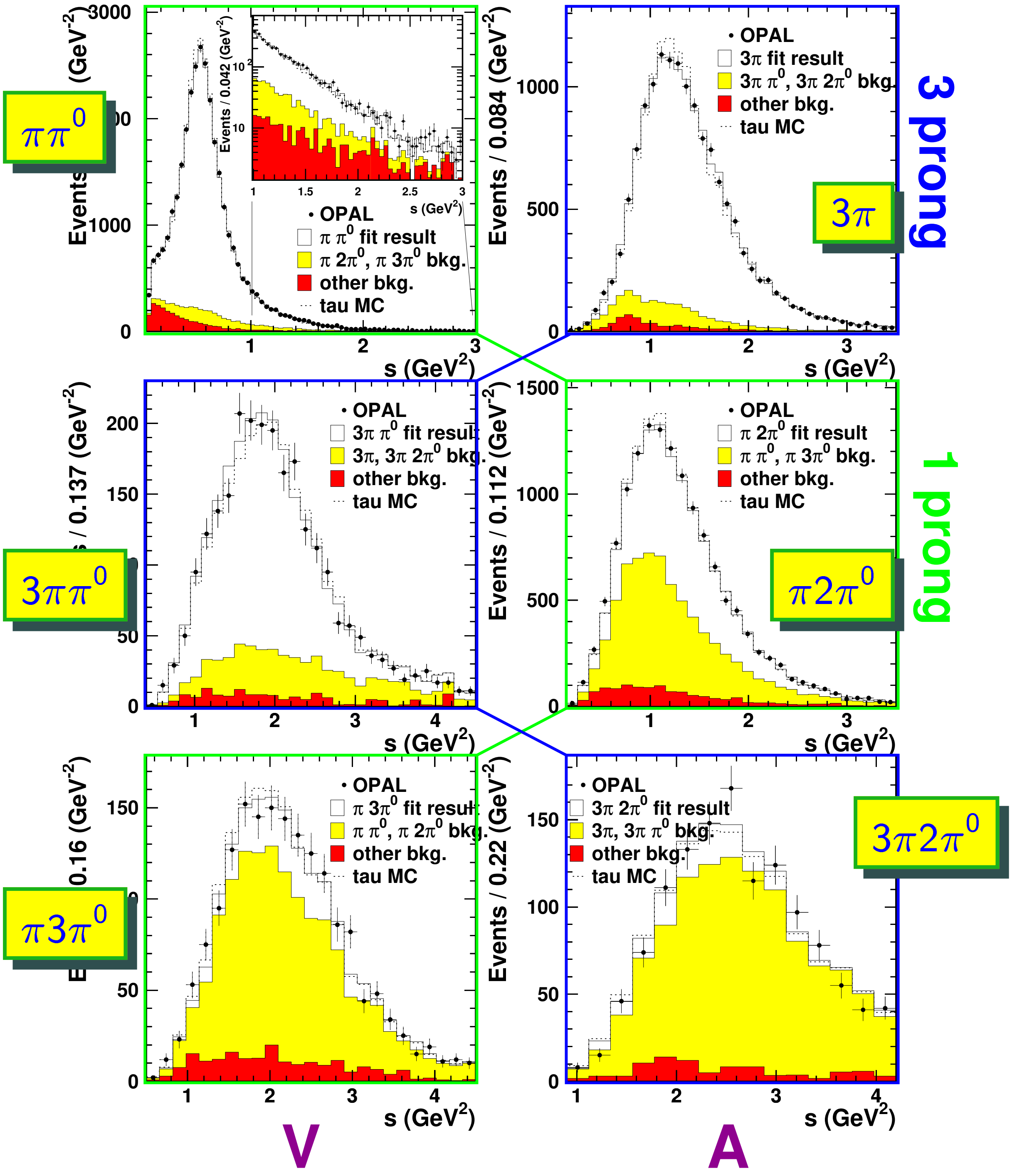
Reconstructed Number of π^0s

$E_{\pi^0}^{\text{1-prong}} > 0.7\,\text{GeV}$

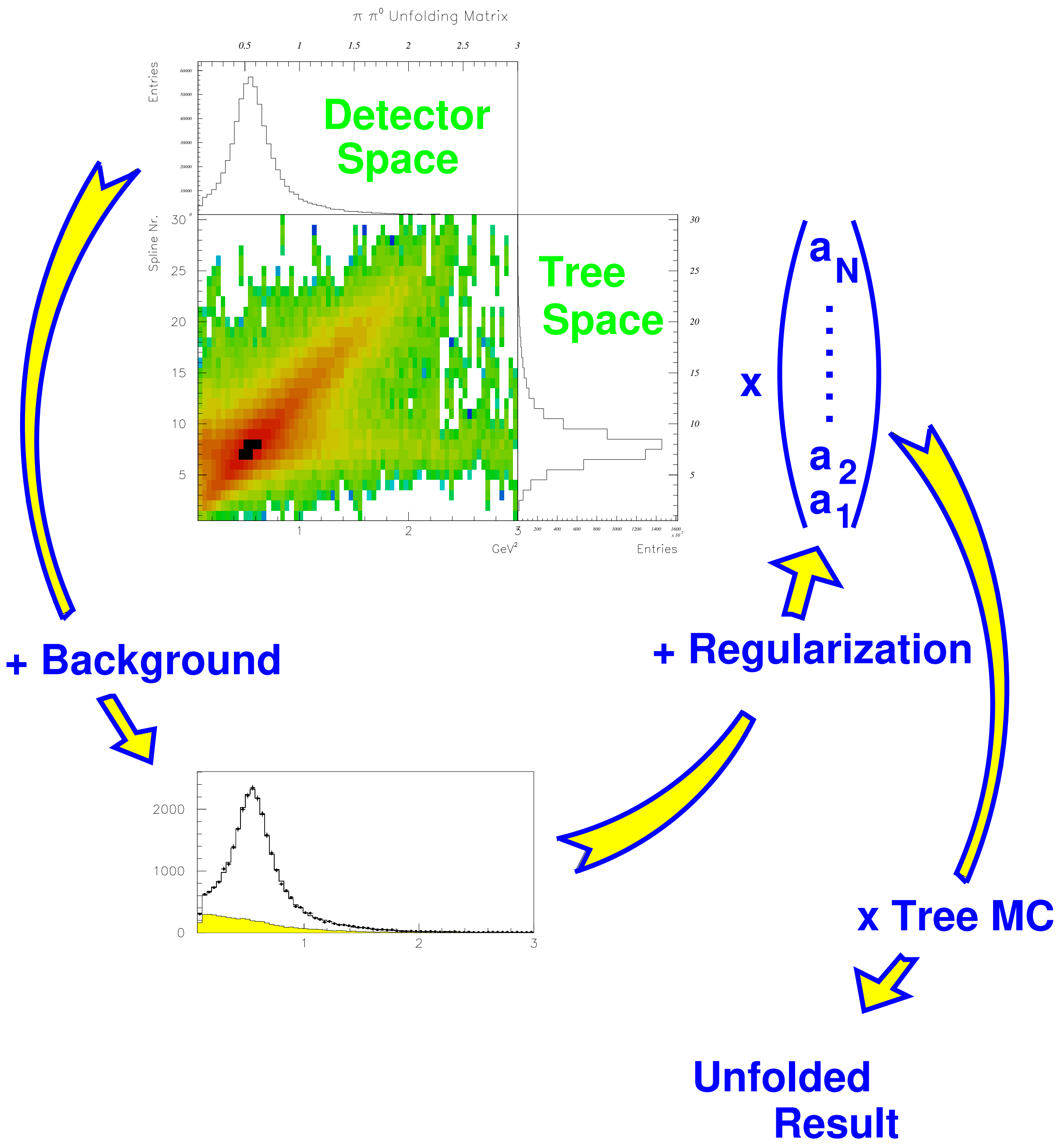
$E_{\pi^0}^{\text{3-prong}} > 2.0\,\text{GeV}$



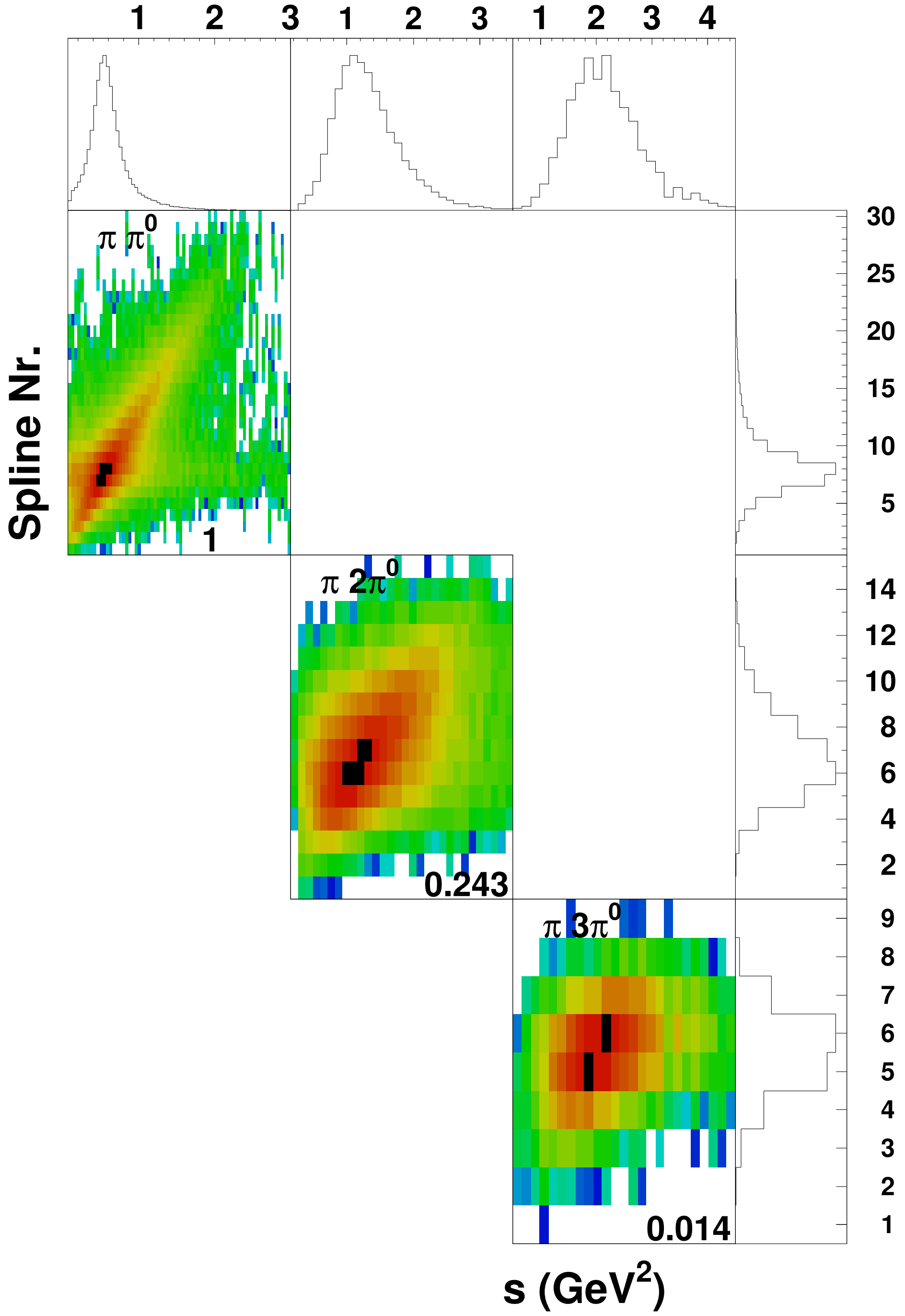
Measured Spectra



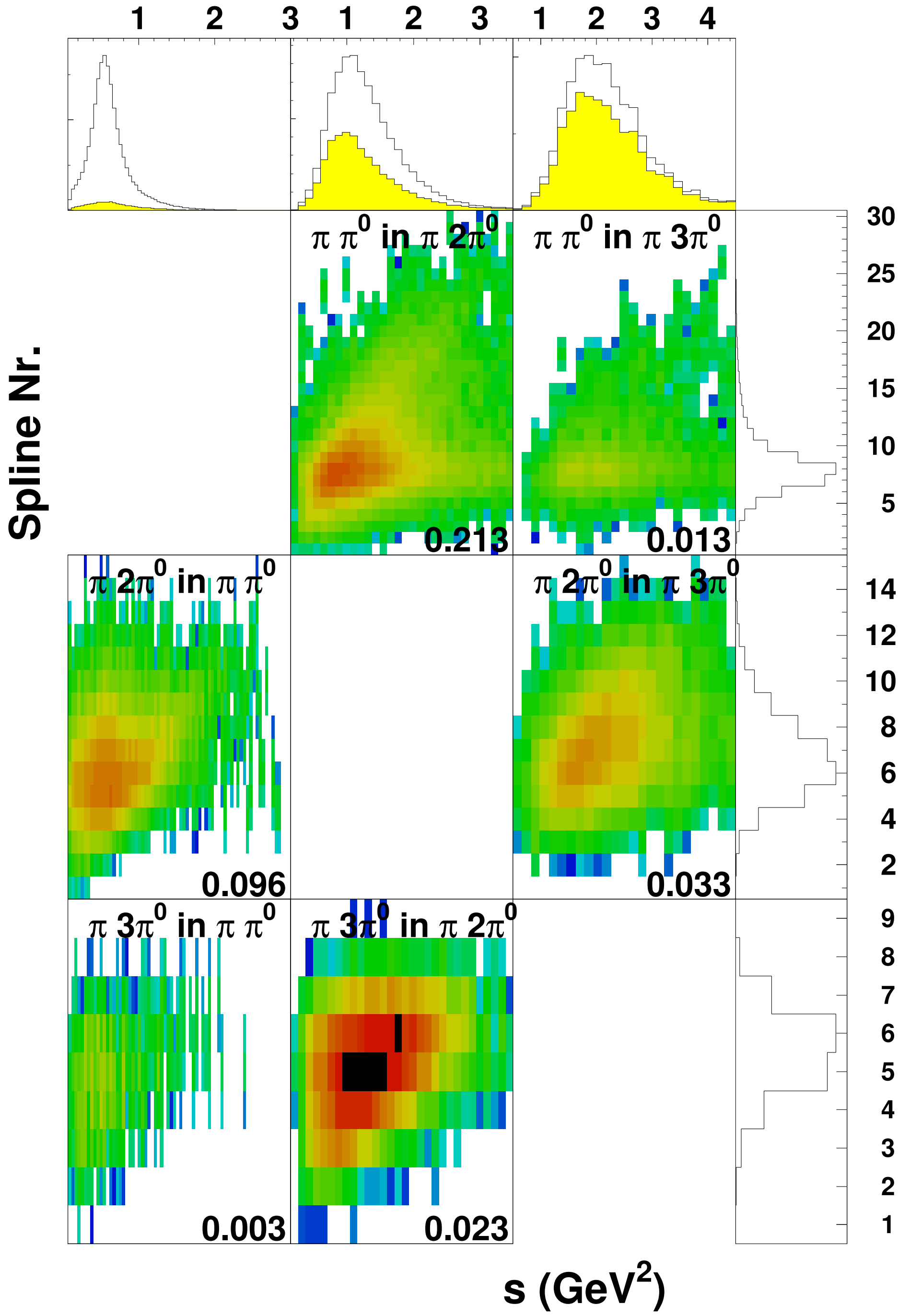
Unfolding Scheme



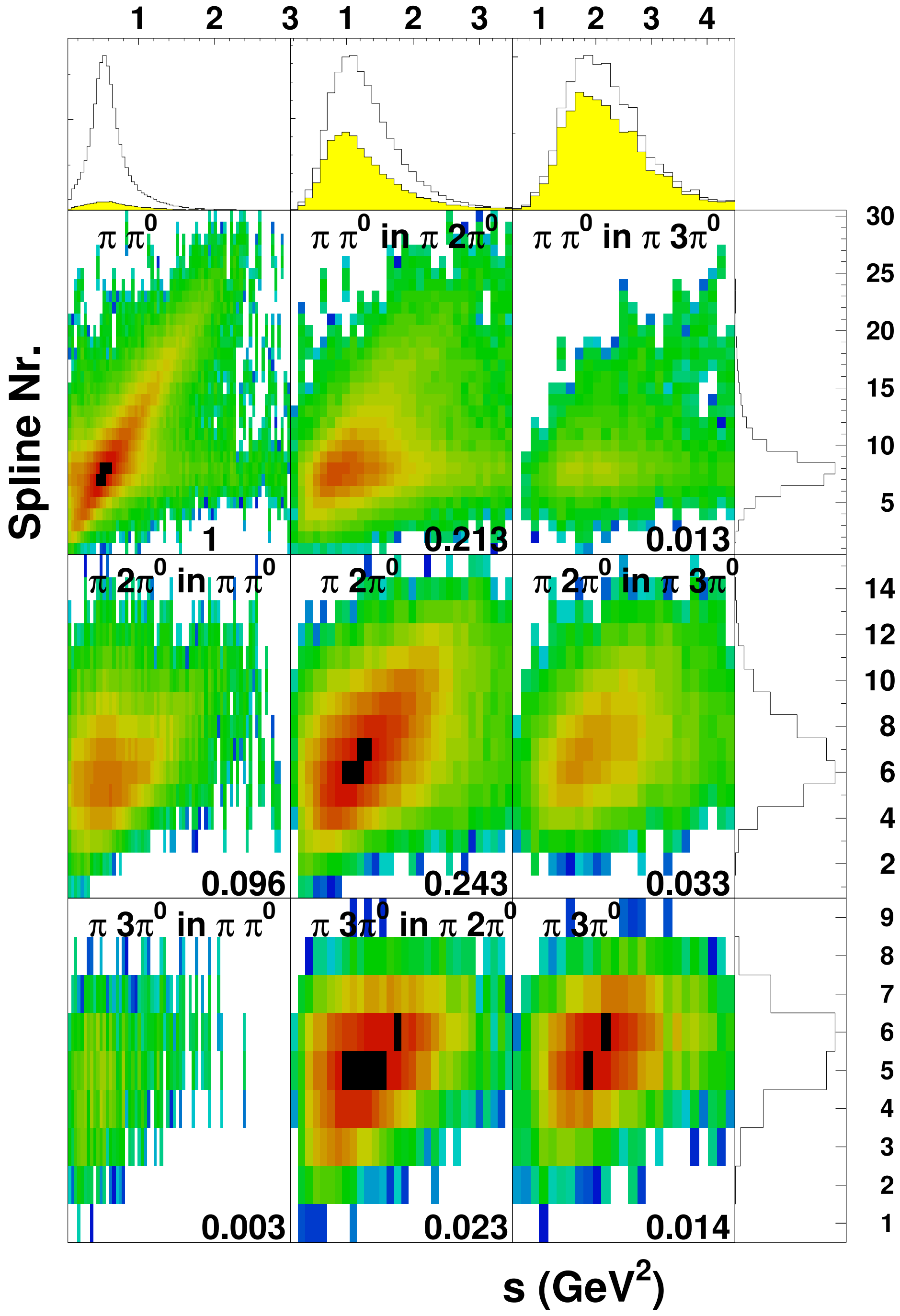
Unfolding simultaneously . . .



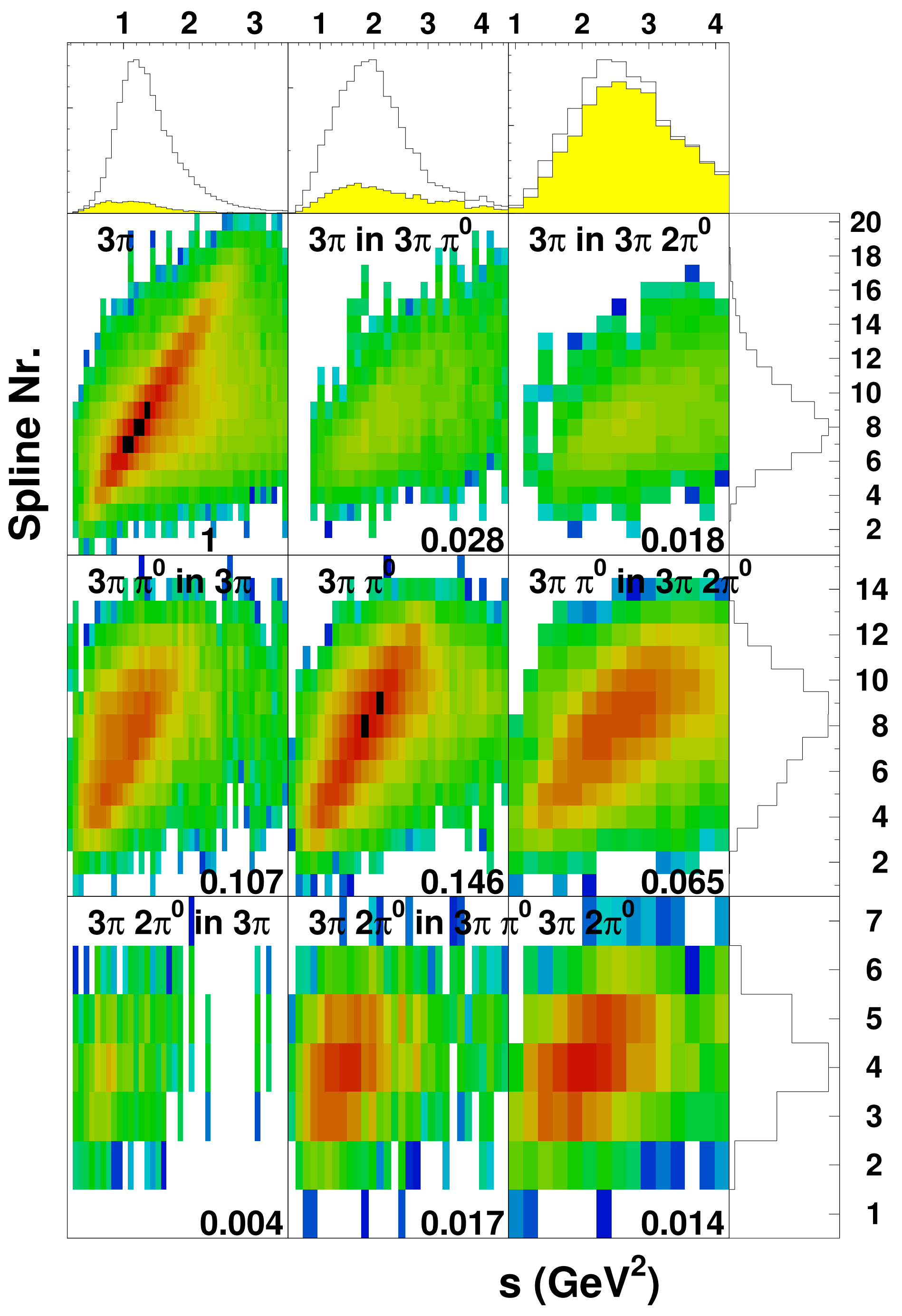
... with Background



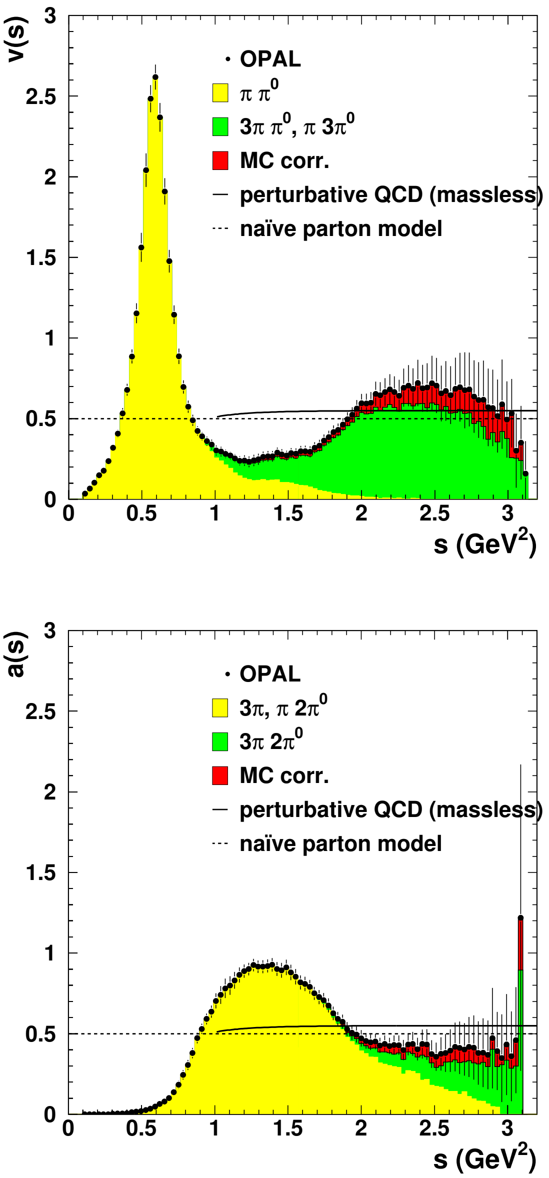
Simultaneous Unfolding of 1-prongs



Simultaneous Unfolding of 3-prongs



Spectral Functions

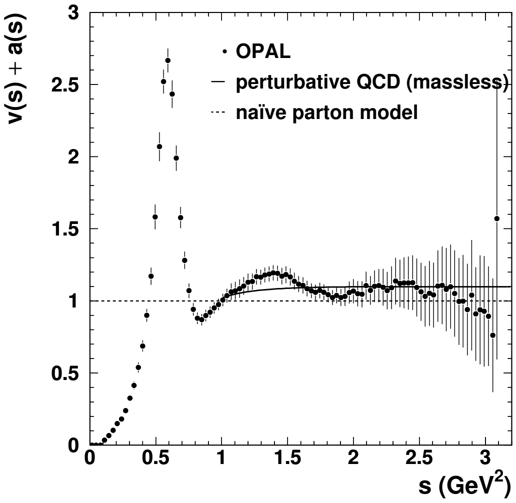


$$v(s) \sim \mathrm{Im}\Pi_V^{(1)}$$

$$a(s) \sim \mathrm{Im}\Pi_A^{(1)}$$

$$v/a(s) \sim \left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1}\sum_{h_{V/A}}\frac{B(\tau\to h_{V/A})}{B(\tau\to e)}\left(\frac{1}{N}\frac{dN}{ds}\right)_{V/A}$$

Applications of the Spectral Functions

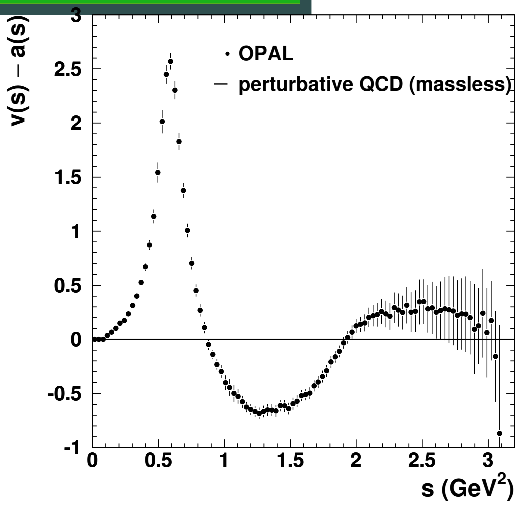


- large cancellation of non-perturbative parts
- good sensitivity to perturbative QCD

kl	$R^{kl}_{\tau,V}$		$R^{kl}_{\tau,A}$	
00	1.764	$\pm$0.016	1.720	$\pm$0.017
10	1.264	$\pm$0.012	1.240	$\pm$0.013
11	0.2980	$\pm$0.0034	0.2510	$\pm$0.0032
12	0.0942	$\pm$0.0019	0.1090	$\pm$0.0019
13	0.0403	$\pm$0.0016	0.0518	$\pm$0.0013

Spectral Moments

$$R^{kl}_{\tau,V/A}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V/A}}{ds}$$



- perturbative parts cancel each other
- sensitive to non-perturbative QCD

Theoretical Description of R_τ

$$\boxed{R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW}\left(1 + \delta_{\text{pert}} + \delta^{V/A}_{\text{mass}} + \delta^{V/A}_{\text{non-pert}}\right)}$$

- perturbative part

$$1+\delta_{\text{pert}} = \oint_{|s|=m_\tau^2} ds \left(1-2\frac{s}{m_\tau^2}+2\frac{s^3}{m_\tau^6}-\frac{s^4}{m_\tau^8}\right) \underbrace{(-s)\frac{d\Pi}{ds}}_{\boxed{D(s)}}$$

CIPT: $D(s) \sim 1+\frac{\alpha_s(-s)}{\pi}+1.64\frac{\alpha_s^2(-s)}{\pi^2}+6.37\frac{\alpha_s^3(-s)}{\pi^3}$

FOPT: $1+\delta_{\text{pert}} = 1+\frac{\alpha_s(m_\tau^2)}{\pi}+5.20\frac{\alpha_s^2(m_\tau^2)}{\pi^2}+26.4\frac{\alpha_s^3(m_\tau^2)}{\pi^3}$

RCPT: $D(s) \sim 1+\sum_{n=1}^{\infty}\kappa_n\beta_0^{n-1}\frac{\alpha_s^n(-s)}{\pi^n}$

- power corrections

OPE: $\delta^{V/A}_{\text{mass}} \sim \frac{m_q^2}{m_\tau^2}$

$$\delta^{V/A}_{\text{non-pert}} \sim C_4^{V/A}\frac{\langle O\rangle^{D=4}}{m_\tau^4}+\underbrace{C_6^{V/A}\frac{\langle O\rangle^{D=6}}{m_\tau^6}}_{\delta^6_{V/A}}+\underbrace{C_8^{V/A}\frac{\langle O\rangle^{D=8}}{m_\tau^8}}_{\delta^8_{V/A}}$$

$\boxed{\langle\alpha_s GG\rangle, \ldots}$

Results of the V+A fit

$\alpha_s(m_\tau^2)$

		exp.	theo.
CIPT:	0.348	± 0.009	± 0.019
FOPT:	0.324	± 0.007	± 0.013
RCPT:	0.306	± 0.005	± 0.011

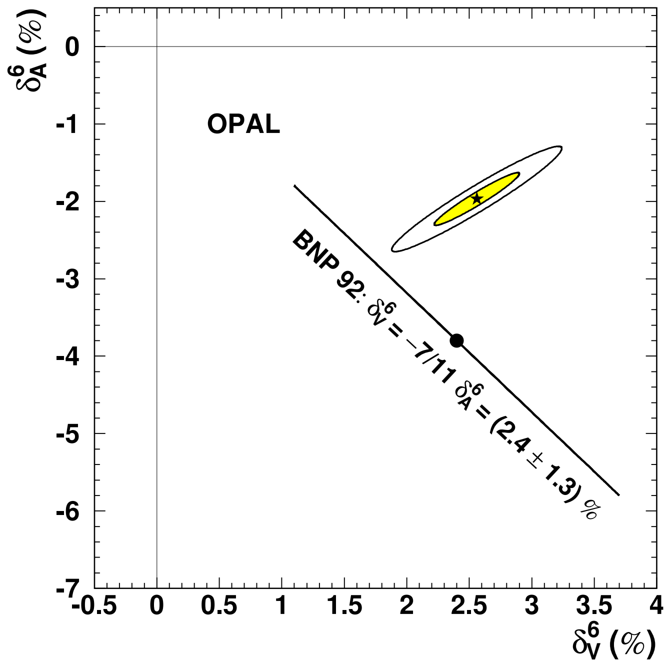
$\Delta\alpha_s(m_\tau^2)_{\mathrm{CIPT}}$

τ Branching Ratios	0.009	
dR_τ/ds	0.002	
E, p Resolution	0.002	$\pm 0.009_{\mathrm{exp.}}$
Monte Carlo	0.001	
Unfolding	<0.0005	
Renormalization Scheme	0.015	
$O(\alpha_s^4)$	0.012	$\pm 0.019_{\mathrm{theo.}}$
Renormalization Scale	0.006	

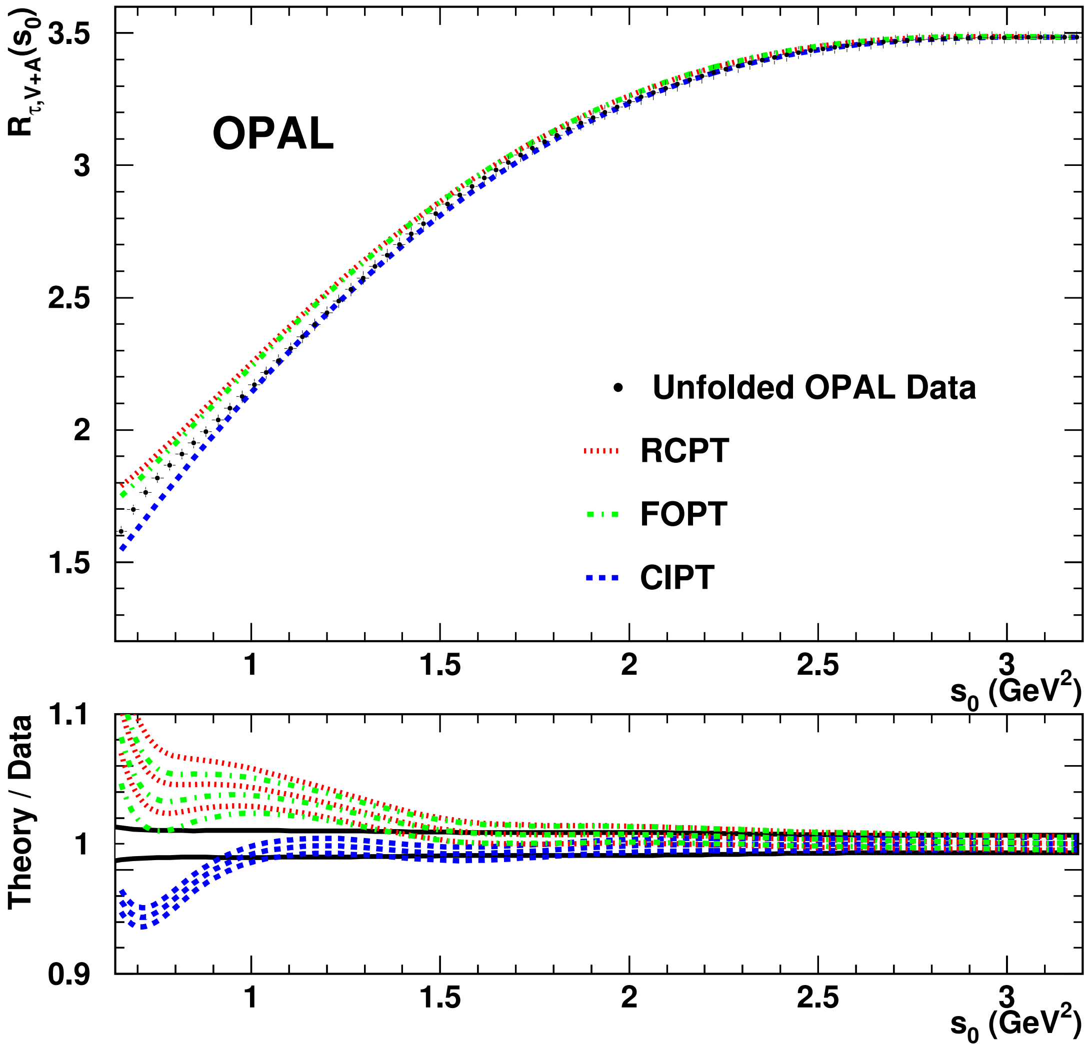
Results of the V&A fit

Non-Perturbative Corrections

		exp.	theo.
$\langle\alpha_s GG\rangle$	(0.001	$\pm$0.008	$\pm$0.004) GeV^4
δ_V^6	(2.56	$\pm$0.34	$\pm$0.06) %
δ_A^6	($-$1.97	$\pm$0.33	$\pm$0.10) %
δ_V^8	($-$0.80	$\pm$0.14	$\pm$0.02) %
δ_A^8	(0.41	$\pm$0.19	$\pm$0.02) %



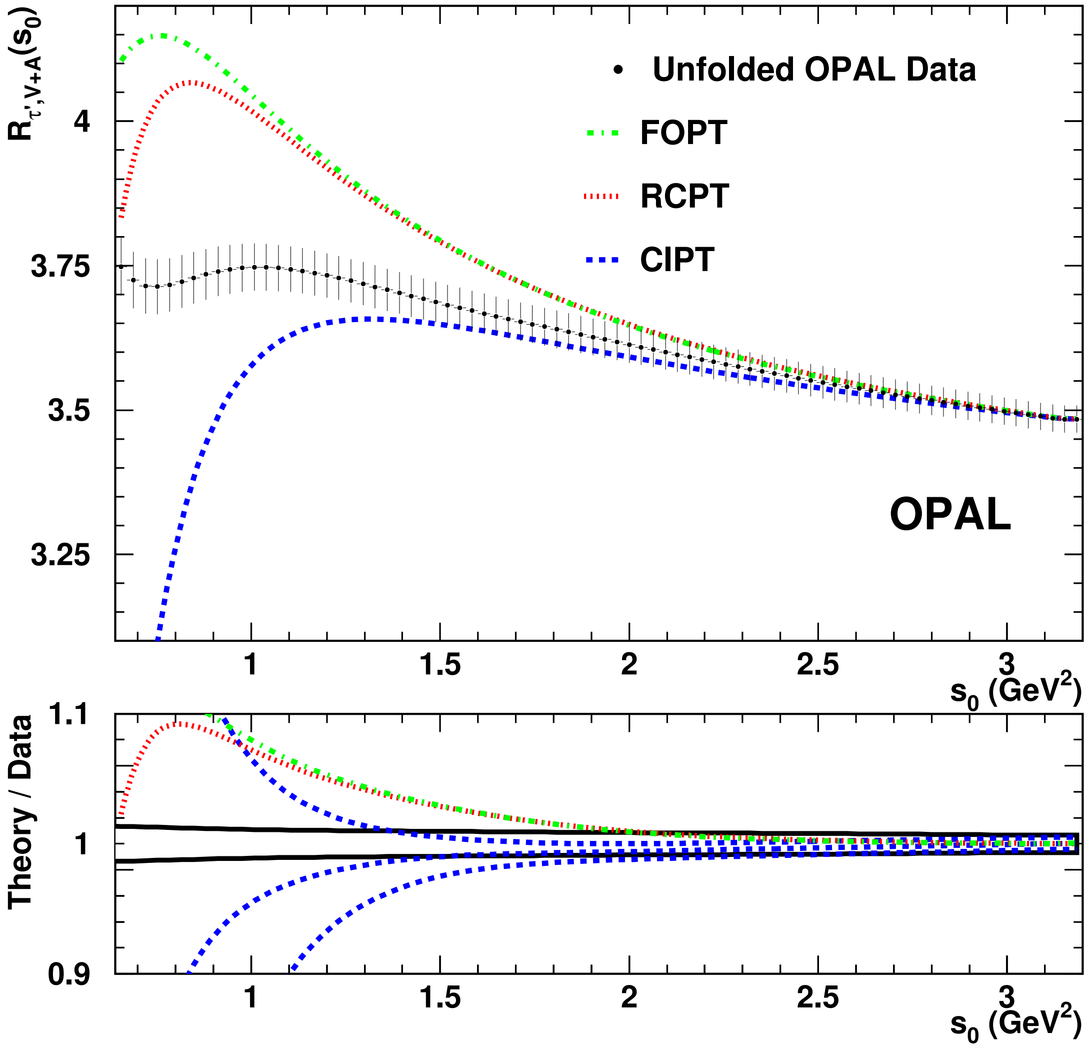
Running $R_{\tau,\mathrm{V+A}}$



$$R_{\tau,\mathrm{V+A}}(s_0) \sim$$

$$\int_0^{s_0} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+\frac{2s}{m_\tau^2}\right)(v(s)+a(s))+a^{(0)}(s)\right]$$

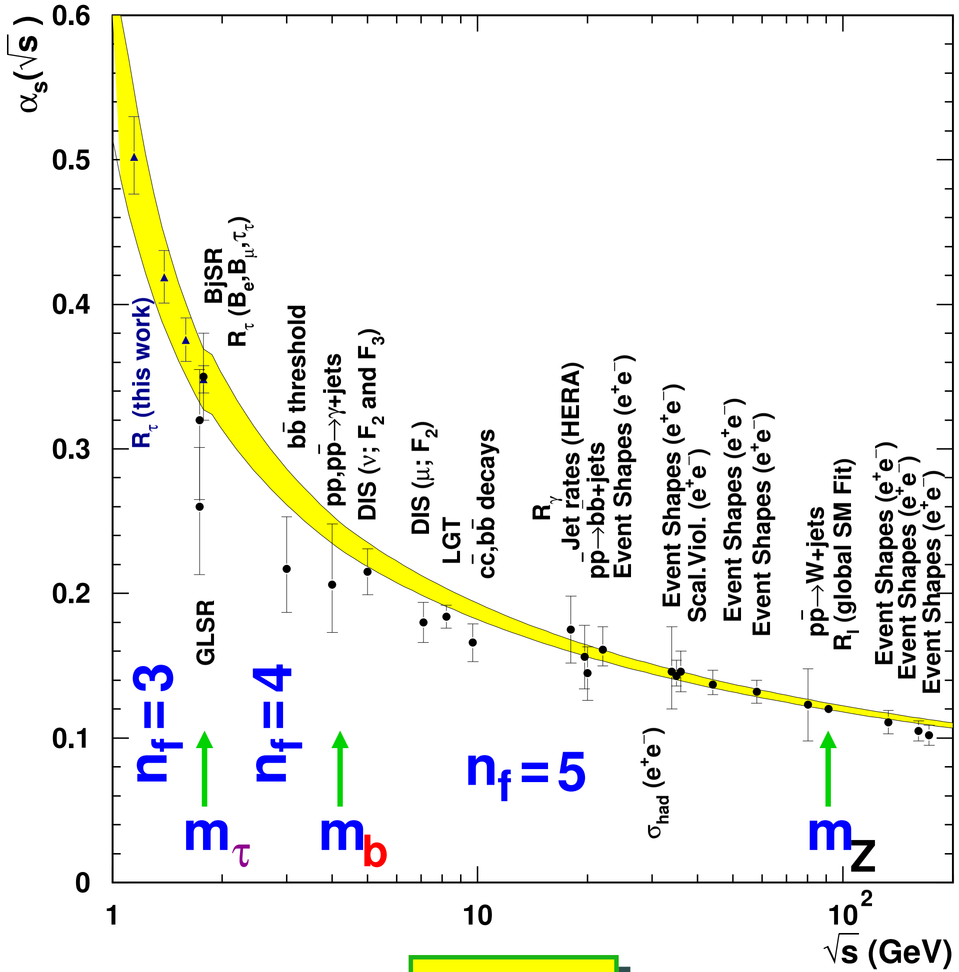
Decay Ratio for a hypothetical τ'



$$R_{\tau',V+A}(s_0 = m^2_{\tau'}) \sim$$

$$\int_0^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2\left[\left(1+\frac{2s}{s_0}\right)(v(s)+a(s))+a^{(0)}(s)\right]$$

Running α_s

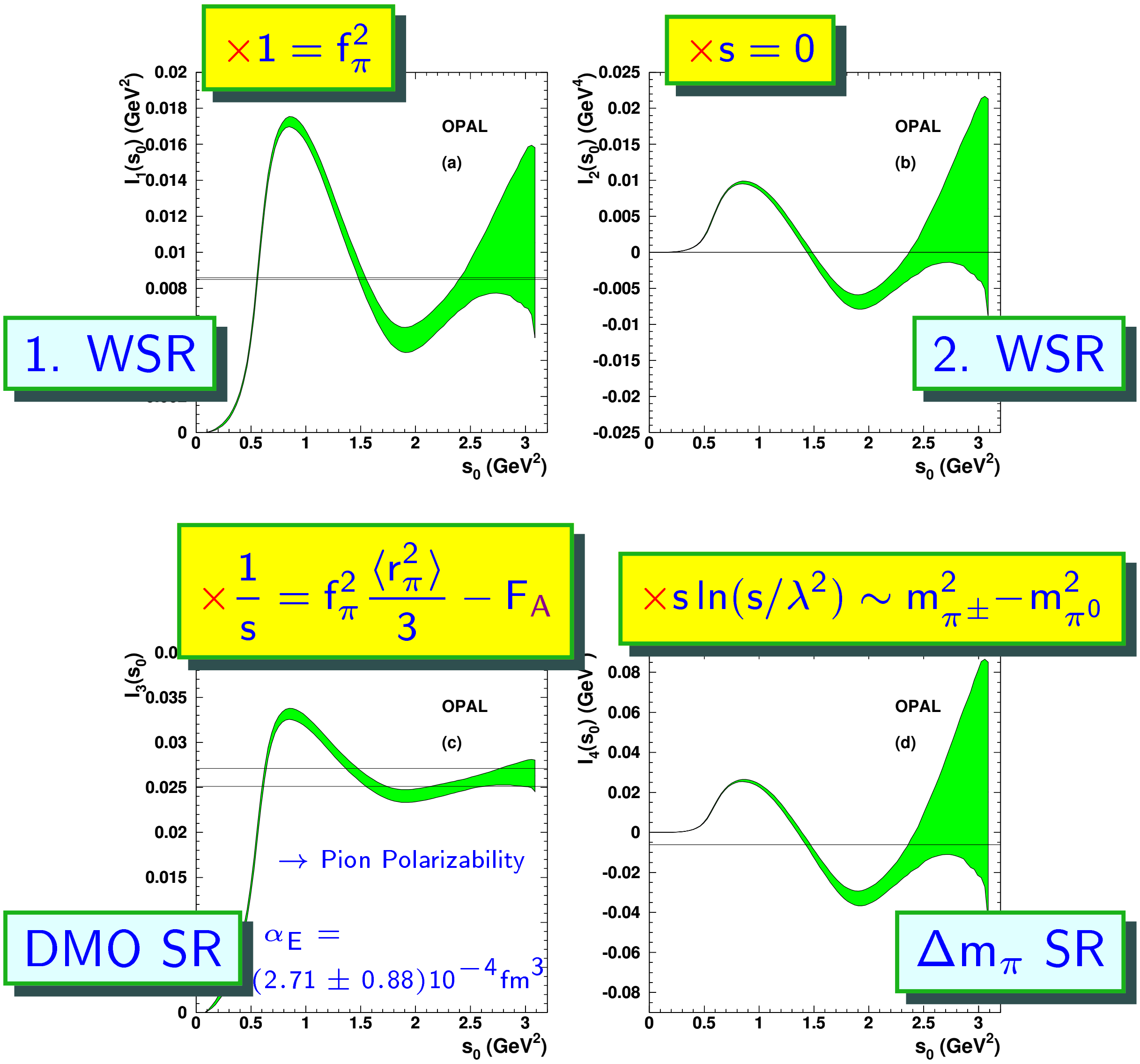


$\alpha_s(m_Z)$

		exp.	theo.	evol.
CIPT:	0.1219	± 0.0010	± 0.0017	± 0.0003
FOPT:	0.1191	± 0.0008	± 0.0013	± 0.0003
RCPT:	0.1169	± 0.0007	± 0.0015	± 0.0003

QCD Sum Rules

$$I_{1,2,3,4}(s_0) = \frac{1}{4\pi^2}\int_0^{s_0} ds\,(v(s) - a(s)) \times$$



Summary

- Measurement of the vector and axial-vector spectral functions
- Simultaneous determination of perturbative (α_s) and non-perturbative QCD parameters (power corrections, $\langle \alpha_\mathrm{s} GG \rangle$) for three different descriptions of the perturbative part
- Test of the OPE with 'running' $R_{\tau,V+A}(s_0)$

  $\rightarrow$ 'Running' α_s
- Test of QCD sum rules