QCD in τ Decays

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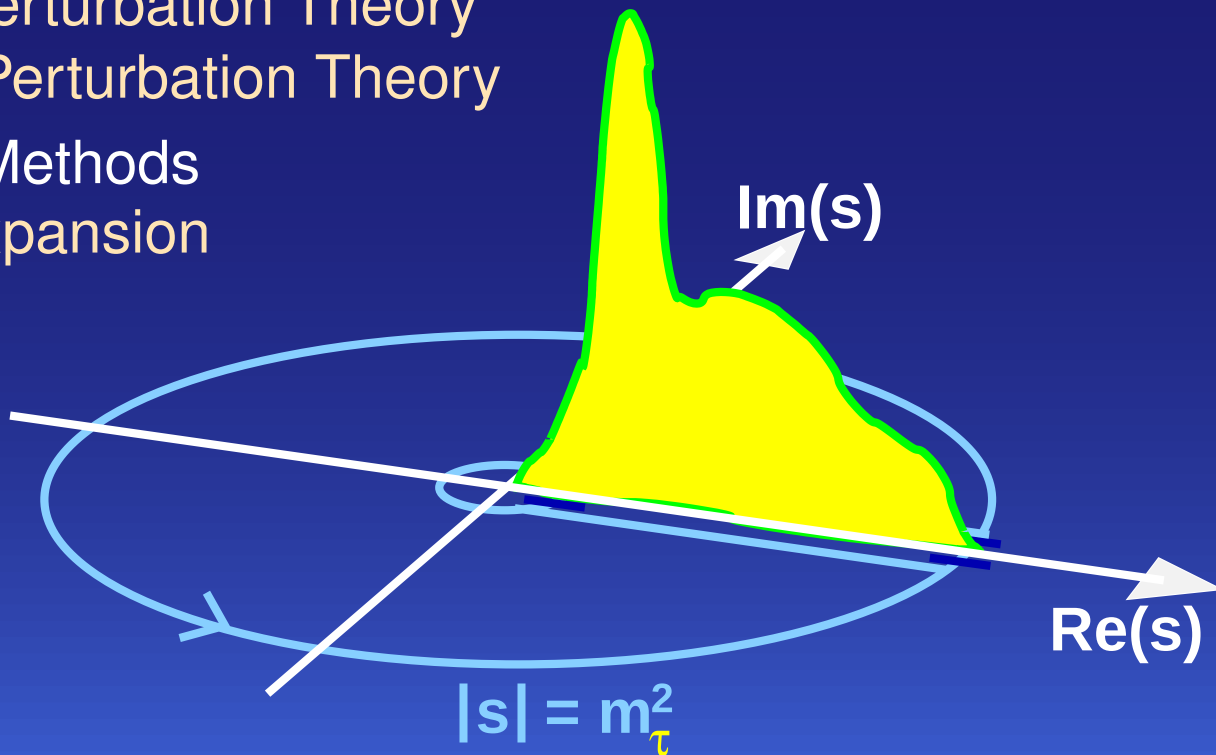
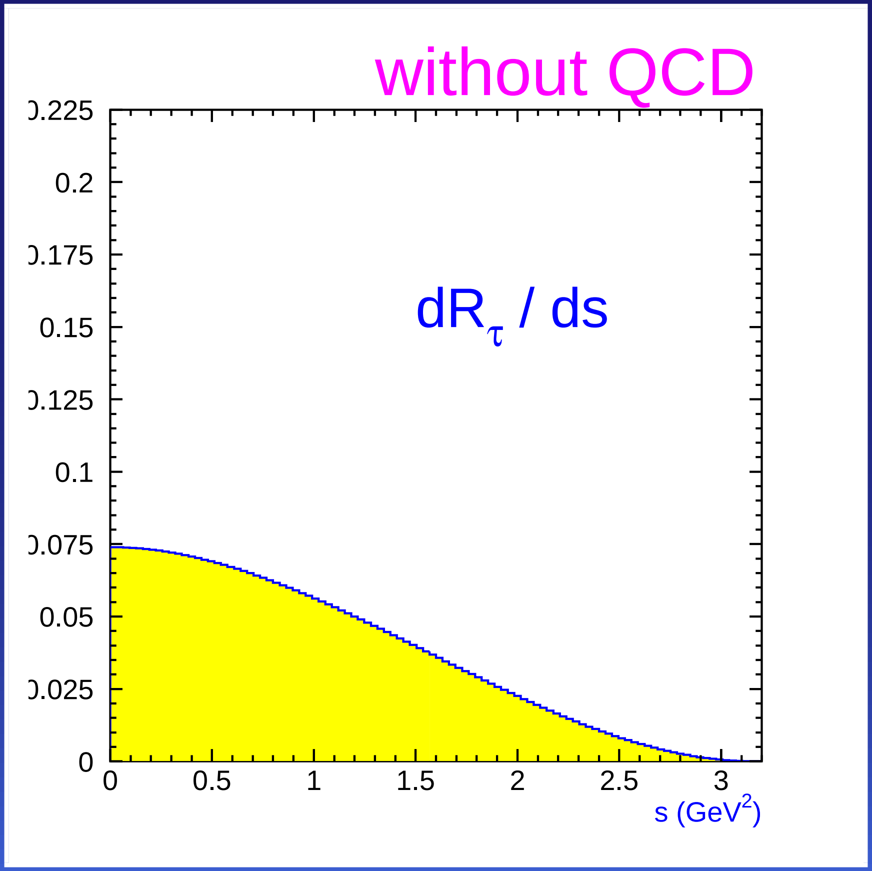
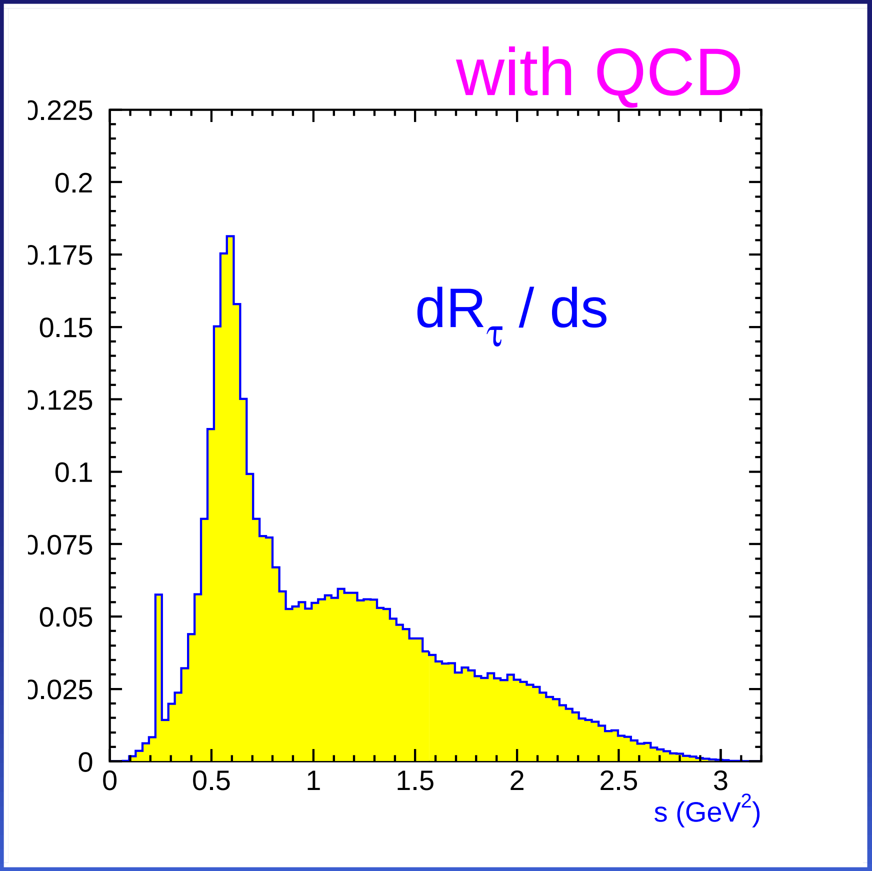
- Hadronic τ Decays
  - How a lepton can be sensitive to QCD
- Perturbative QCD Methods
  - Fixed Order Perturbation Theory
  - Contour Improved Perturbation Theory
  - Renormalon Chain Perturbation Theory
- Non-Perturbative QCD Methods
  - Operator Product Expansion
- Measurements
  - Branching Ratios
  - Spectral Functions
- QCD Fits



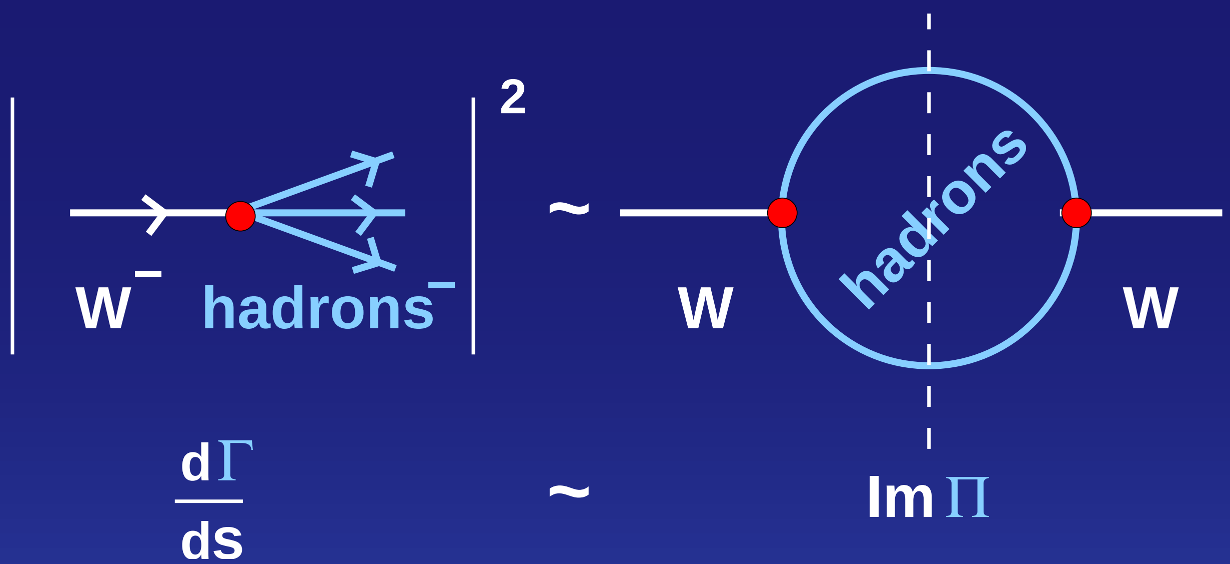
- The τ-lepton with $m_\tau = 1.777\,\mathrm{GeV}$ is the only lepton heavy enough to decay into hadrons
- The hadronic decay ratio of the τ is defined as:
  $$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \text{ hadrons})}{\Gamma(\tau \to \nu_\tau\, \mathrm{e}\, \bar{\nu}_\mathrm{e})}$$
- the naive expectation is just the number of colors:
  $$R_\tau = N_C\,(|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2) = 3$$
- experimentally one finds a 20 % larger value: $R_\tau = 3.635$

- The hadronic decay spectrum of the τ, dR_τ/ds i.e. the spectrum of the squared masses of the hadrons the τ decays into shows this sensitivity
- perturbative QCD is repsonsible for the total increase of 20 %
- non-perturbative QCD is repsonsible for the observed resonance structure



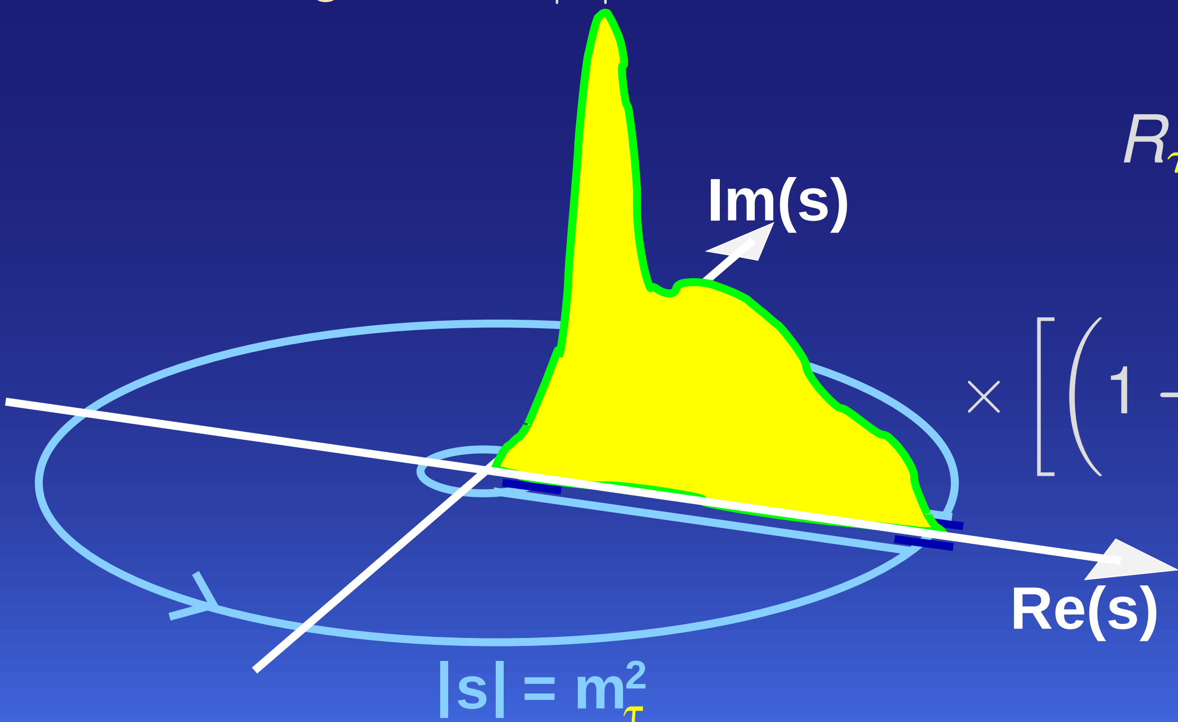


- Use the optical theorem:
  - The differential decay width $\mathrm{d}\Gamma/\mathrm{d}s$ for the τ going into hadrons is proportional to the imaginary part of the vacuum polarization $\mathrm{Im}\Pi$ (also called spectral function) of the W propagator
  - this means it is enough to calculate the inclusive vacuum polarization of the W instead of every single final state of the hadronic τ decays



$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{\mathrm{d}s}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

- Use the Cauchy theorem:
  - since Π is analytic (except for the real positive s-axis where it might have poles) in the entire complex s-plane
  - and due to the identity $\mathrm{Im}\Pi(s_+ + i\epsilon) = \frac{1}{2i}(\Pi(s_+ + i\epsilon) - \Pi(s_+ - i\epsilon))$
  - the integral along the real positive s-axis can be expressed as a circular integral at $|s| = m_\tau^2$
  - this means all QCD calculations are done at a well defined rather high scale $|s| = m_\tau^2$ over the vacuum polariztion amplitude



$$R_\tau = 6\pi i \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[\left(1 + 2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$

- All the dynamics of the vacuum polarization amplitude is in its logarithmic derivative, the so-called Adler-function: $D(s) = -s\frac{d\Pi(s)}{ds}$
- the Adler-function can be written as polynomial in the strong coupling constant α_s: $D(s) = \frac{1}{4\pi^2}\sum_{n=0}^{\infty} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$, where the K_n are known up to $n = 3$ and K_4 is partially known:

$$
\begin{aligned}
K_0 &= 1 \\
K_1 &= 1 \\
K_2 &= \frac{299}{24} - 9\zeta(3) = 1.63982\ldots \\
K_3 &= \frac{58057}{288} - \frac{779}{4}\zeta(3) + \frac{75}{2}\zeta(5) = 6.37101\ldots \\
K_4 &\simeq 25 \pm 25
\end{aligned}
$$

- convert the contour integral over $\Pi(s)$ via partial integraion in a contour integral over $D(s)$

$$\oint_{|s|=m_\tau^2} \mathrm{d}s\, g(s)\,\Pi(s) = \oint_{|s|=m_\tau^2} \frac{\mathrm{d}s}{s}\left[G(s)-G(m_\tau^2)\right](-s)\frac{\mathrm{d}}{\mathrm{d}s}\Pi(s)$$

$$= \oint_{|s|=m_\tau^2} \frac{\mathrm{d}s}{s}\left[G(s)-G(m_\tau^2)\right]D(s)$$

- where the polynomial $g(s) = 6\pi i \frac{1}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+2\frac{s}{m_\tau^2}\right)$
- has the antiderivative $G(s) = 3\pi i\left(2\frac{s}{m_\tau^2} - 2\frac{s^3}{m_\tau^6} + \frac{s^4}{m_\tau^8}\right)$
- note that $g(s)$ describes the $J=1$ part only since for massless quarks the scalar and pseudo-scalar parts vanish

$$R_\tau = 3\sum_{n=0}^{4}\frac{K_n}{2\pi i}\oint_{|s|=m_\tau^2}\frac{\mathrm{d}s}{s}\left(1-2\frac{s}{m_\tau^2}+2\frac{s^3}{m_\tau^6}-\frac{s^4}{m_\tau^8}\right)\left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

- the perturbative descriptions of hadronic τ decays start all with the same integral given on the previous slide
- they differ in the way the integral is calculated
- most interesting part is the treatment of $\alpha_s(-s)$ on the circle $|s| = m_\tau^2$
- QCD does not tell us how large $\alpha_s(\mu^2)$ at a given scale μ is, but QCD does tell us what $\alpha_s(\mu_1^2)$ at some scale μ_1 is if we know it at some other scale μ_0
- this prediction is made with the so-called β-function:

$$\beta(a_s) = \mu^2 \frac{\mathrm{d}a_s}{\mathrm{d}\mu^2} = \beta_0\, a_s^2 + \beta_1\, a_s^3 + \beta_2\, a_s^4 + \beta_3\, a_s^5 + O(a_s^6), \text{ with } a_s = \frac{\alpha_s(\mu^2)}{4\pi},$$

and where the β_n are known up to $n = 3$:

$$\beta_0 = -11 + \frac{2}{3} n_f, \qquad \beta_1 = -102 + \frac{38}{3} n_f, \qquad \beta_2 = -\frac{2857}{2} + \frac{5033}{18} n_f - \frac{325}{54} n_f^2,$$

$$\beta_3 = -\frac{149753}{6} - 3564\zeta(3) + \left(\frac{1078361}{162} + \frac{6508}{27}\zeta(3)\right) n_f - \left(\frac{50065}{162} + \frac{6472}{81}\zeta(3)\right) n_f^2 - \frac{1093}{729} n_f^3.$$

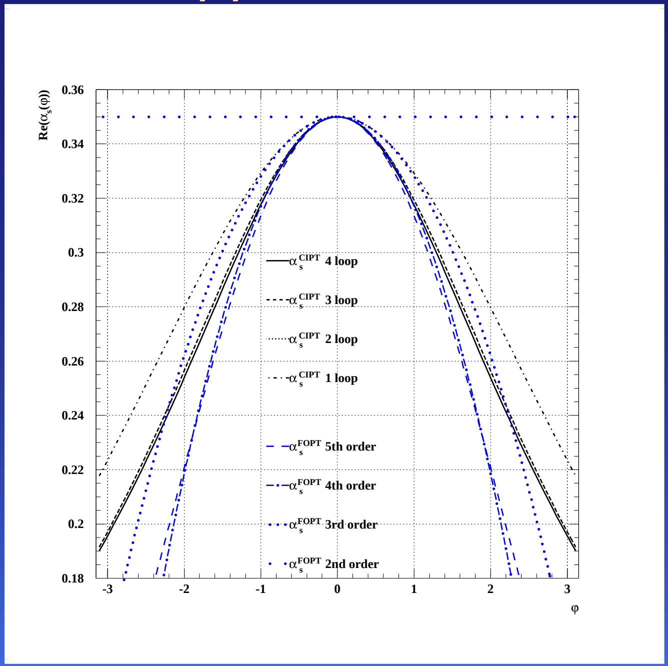
- The Adler-function shown used K_n for a fixed choice of renormalization scale $-\mu^2 = m_\tau^2$
- Physics should not depend on this choice
- The truncation of the perturbative series introduces a residual dependency: $\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D(s, \mu^2) = 0 + O\left(\alpha_s^5(\mu^2)\right)$
- This can be solved for each order with the β-function ($\eta = \ln(-s/\mu^2)$):

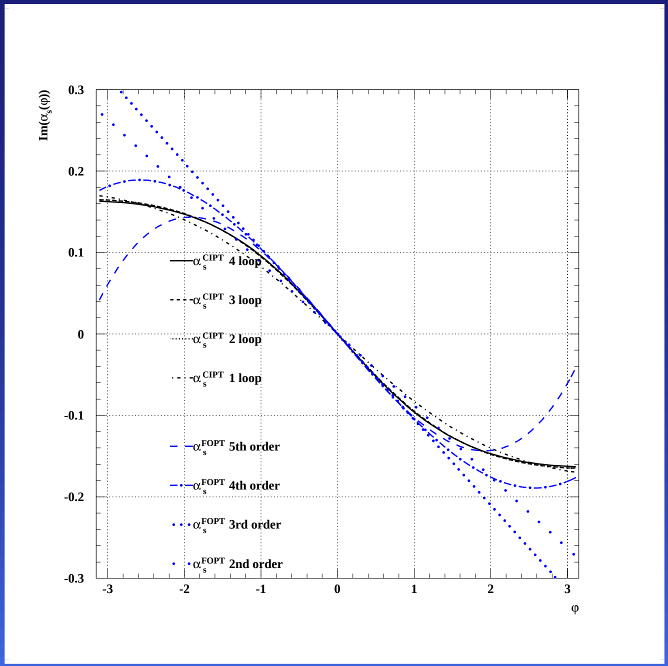
$$
\begin{aligned}
K_0(\eta) &= K_0 \\
K_1(\eta) &= K_1 \\
K_2(\eta) &= K_2 + \frac{\beta_0}{4}\eta \\
K_3(\eta) &= K_3 + \left(\frac{\beta_1}{16} + \frac{\beta_0}{2}K_2\right)\eta + \frac{\beta_0^2}{16}\eta^2 \\
K_4(\eta) &= K_4 + \left(\frac{\beta_2}{64} + \frac{\beta_1}{8}K_2 + \frac{3\beta_0}{4}K_3\right)\eta + \left(\frac{5\beta_1\beta_0}{128} + \frac{3\beta_0^2}{16}K_2\right)\eta^2 + \frac{\beta_0^3}{64}\eta^3
\end{aligned}
$$

- Fixed Order Perturbation Theory
  - make Taylor-expansion of $\alpha_s(-s)$ on the circle $|s| = m_\tau^2$ around $\alpha_s(m_\tau^2)$
  - insert Taylor-expansion in the integral which becomes solvable in all orders
  - order the result in powers of $\alpha_s(m_\tau^2)$
  - keep only the terms up to a fixed order in $\alpha_s(m_\tau^2)$

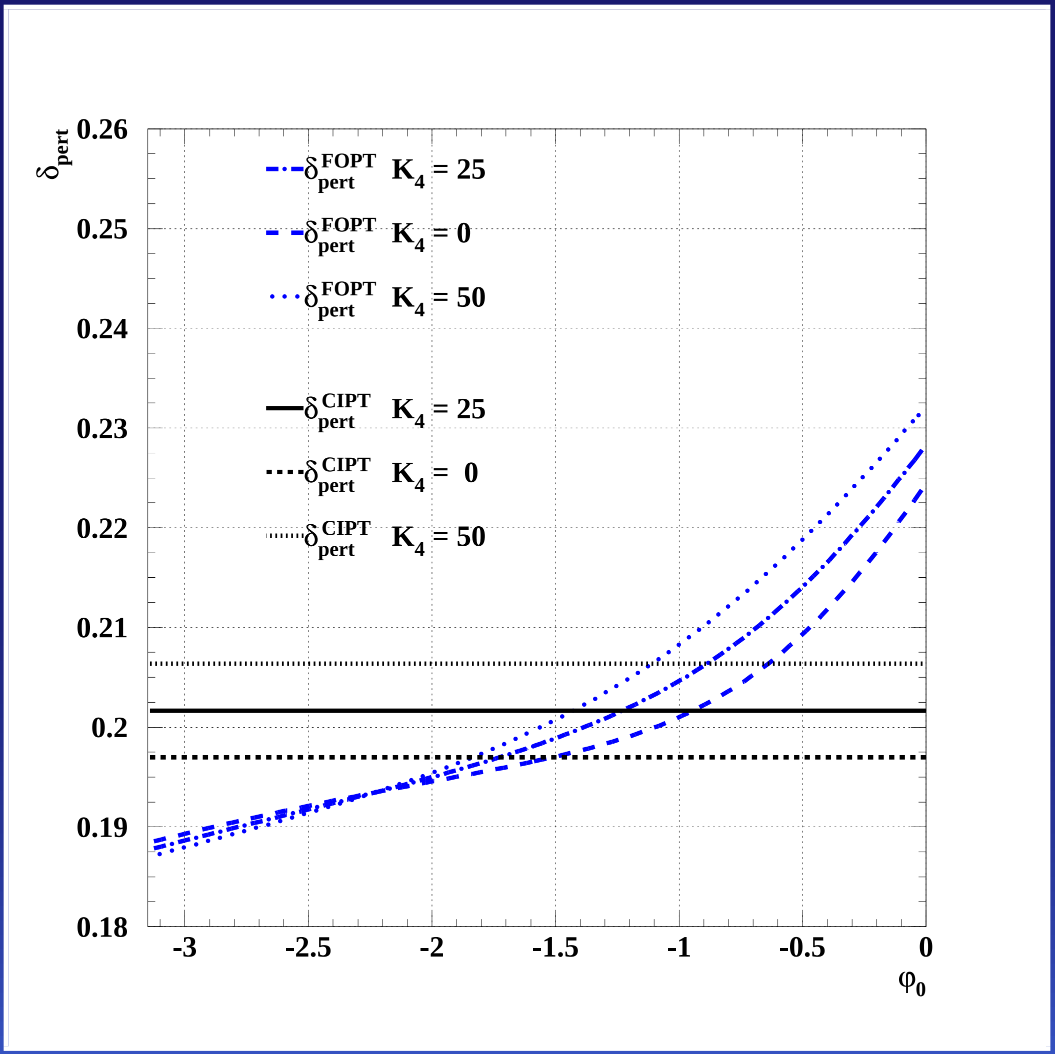
$$
\begin{aligned}
R_\tau &= 3\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2023\ldots\frac{\alpha_s^2(m_\tau^2)}{\pi^2} + 26.3659\ldots\frac{\alpha_s^3(m_\tau^2)}{\pi^3}\right.\\
&\quad + (K_4 + 78.0029\ldots)\frac{\alpha_s^4(m_\tau^2)}{\pi^4}\\
&\quad \left. + (K_5 + 14.25\,K_4 - 391.542\ldots)\frac{\alpha_s^5(m_\tau^2)}{\pi^5} + O(\alpha_s^6)\right)
\end{aligned}
$$

- **C**ontour **I**mproved **P**erturbation **T**heory
  - evolve $\alpha_s(-s)$ in small steps numerically using the β-function on the circle $|s| = m_\tau^2$
  - insert the numerical values in the integral and solve it numerically too
  - the result contains integrals over terms of the form
    $$s^k \left[\left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^m \ln^n \frac{-s}{m_\tau^2} \right]^l, \text{ with } k = 0, 1, 3, 4;\ l = 1, \ldots, 4;\ m > 4/l$$
    (for FOPT up to 4th order), and $n = 1, \ldots, m - 1$, which are neglected in the FOPT approach
- plots (real part: left, imaginary part: right) show Taylor-expanded $\alpha_s(s = m_\tau^2 \exp(i\varphi))$ (blue) and numerical result (black) for various orders





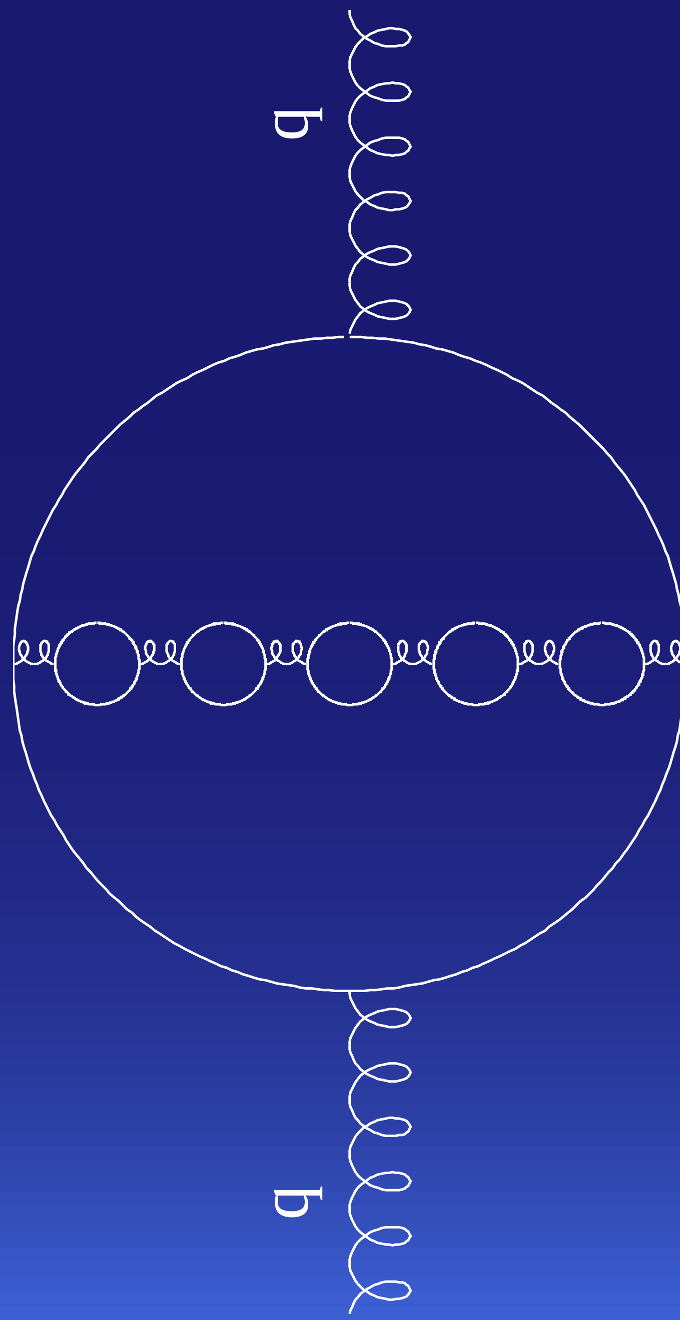
- One could compare CIPT and FOPT by simply developing FOPT around a different point on the circle $s = m_\tau^2 \exp(i\varphi)$ instead of the usual point $\varphi_0 = 0$
- Use $\alpha_s(m_\tau^2 \exp(-i\varphi_0)) = \alpha_s(m_\tau^2 \exp(i\varphi_0))^*$ to split the FOPT integral into 2 half-circle integrals with Taylor-expansions around φ_0 and $-\varphi_0$, respectively
- Plot shows $\delta_{\rm pert}$, the perturbative part of R_τ for generalized FOPT and CIPT as a function of φ_0
- Intrinsic uncertainty for FOPT $3.5\times$ larger than uncertainty from K_4 on CIPT
- Average from FOPT agrees with CIPT



- Renormalon Cchain Perturbation Theory
  - in the limit that all $\beta_{k,k>0}$ vanish
  - and only the terms with the highest power of β_0 are kept
  - one can re-write the K_n of the Adler-function as power series in β_0
  - keeping only the largest power in β_0 one can re-sum the Adler-function to all orders in α_s up to some Renormalon ambguities

$$\triangle D^{(n)} \propto \frac{\Lambda^{2n}}{s^n}, n = 2, 3, \ldots$$

  - this method corresponds to an insertion of fermion loops in the gluon-propagator
- RCPT is usually combined with FOPT up to the 3rd order in order to include some known $\beta_{k,k>0}$ and non-leading β_0 terms

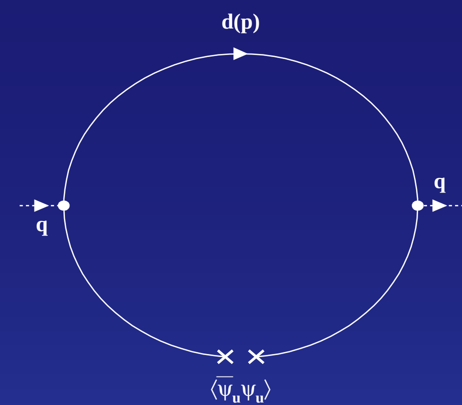


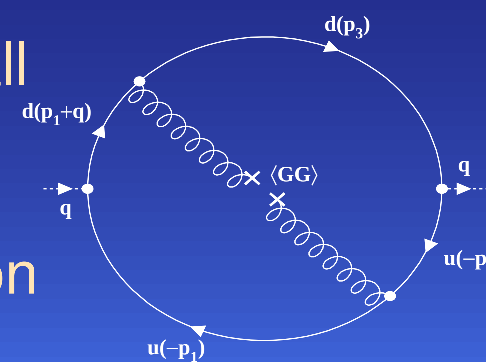
► Operator Product Expansion

- the Adler-function describes the perturbative part of the vacuum-polarization only
- systematic way to separate perturbativ short-distance effects from non-perturbative long-distance effects is given by the OPE:

$$\Pi(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim(O)=D} C(s,\mu)\langle O(\mu)\rangle$$

- $C(s,\mu)$ are perturbative factors (Wilson-coefficients)
- $\langle O(s,\mu)\rangle$ are kondensates (vacuum-expectation values) of local operators and contain the non-perturbative parts
- $D = 0$ corresponds to the perturbative part for massless quarks
- $D = 2$ corresponds to quark-mass corrections (small for non-strange τ-decays)
- $D = 4, 6, 8, \ldots$ are the dimensions for the so-called power corrections with non-trivial vacuum-expecation values of the operators $\langle O\rangle$





- use different weighting polynomials $p^{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l$ to project out different parts of the power corrections (Cauchy)
- usually 5 moments $kl = 00, 10, 11, 12, 13$ are used to constrain 4 variables $\alpha_s(m_\tau^2)$, δ^4, δ^6, δ^8.

$$\delta^{D,kl} = 8\pi^2 \sum_{\dim(O)=D} \frac{C(\mu)\langle O(\mu)\rangle}{m_\tau^D} \begin{pmatrix} 1 & 0 & -3 & -2 & 0 \\ 1 & 1 & -3 & -5 & -2 \\ 0 & -1 & -1 & 3 & 5 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$$

$D=2$	4	6	8	10	kl
1	0	−3	−2	0	00
1	1	−3	−5	−2	10
0	−1	−1	3	5	11
0	0	1	1	−3	12
0	0	0	−1	−1	13

- contributions for $D > 8$ do not contribute to R_τ
- the $D = 2$ quark-mass corrections are purely perturbative again and can be calculated

$$R_\tau^{kl} = \int_0^{m_\tau^2} \mathrm{d}s \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{\mathrm{d}R_\tau}{\mathrm{d}s} = 3\left(1 + \delta_{\text{pert}}^{kl} + \sum_{D=2,4,6,8,\ldots} \delta^{D,kl}\right)$$

Measurements

- From the discussion of the theory the following requirements for the measurements emerge
  - We need the total decay rate R_τ for the perturbative part
  - We need the spectrum $\mathrm{d}R_\tau/\mathrm{d}s$ to define the moments for the non-perturbative parts
  - since the mass of the strange quark is not small it is best to restrict the measurments to the non-strange decays of the τ
  - To enhance the tests of non-perturbative QCD one can even separate axial-vector (A) and vector (V) decays
  - All perturbative predictions ($D = 0$) remain the same but get in addition a factor $|V_{\mathrm{ud}}|^2$ for non-strange decays and a factor $1/2$ for the separation into $\mathrm{V/A}$
  - The non-perturbative power corrections ($D > 0$) receive the same factors as above but also the $\delta^D_{\mathrm{V/A}}$ will differ

- The total hadronic decay ratio can be predicted from the leptonic branching ratios: $R_\tau = \frac{1 - B_e - B_\mu}{B_e}$
- using lepton-universality B_μ and B_e can be used to predict each other: $B_\mu = \frac{\Gamma_\mu}{\Gamma_e} B_e, \ \frac{\Gamma_\mu}{\Gamma_e} = 0.9726$
- and again asuming lepton-universality the τ lifetime can be used to predict the hadronic decay ratio too: $R_\tau = \frac{1}{\Gamma_e}\frac{1}{\tau_\tau} - 1 - \frac{\Gamma_\mu}{\Gamma_e}$, $\Gamma_e = 4.0329 \cdot 10^{-13}\,\mathrm{GeV}$
- PDG 2004:

$$\begin{aligned} \tau_\tau &= (290.6 \pm 1.1) \times 10^{-15}\,\mathrm{s} \\ B_e &= (17.84 \pm 0.06)\,\% \\ B_\mu &= (17.36 \pm 0.06)\,\% \end{aligned}$$

$$R_\tau = 3.635 \pm 0.012$$

Measurements ► Hadronic Branching Ratios

- 7 most important non-strange decay modes
- odd number of pions: axial-vector (A)
- even number of pions: vector (V)
- PDG 2004:

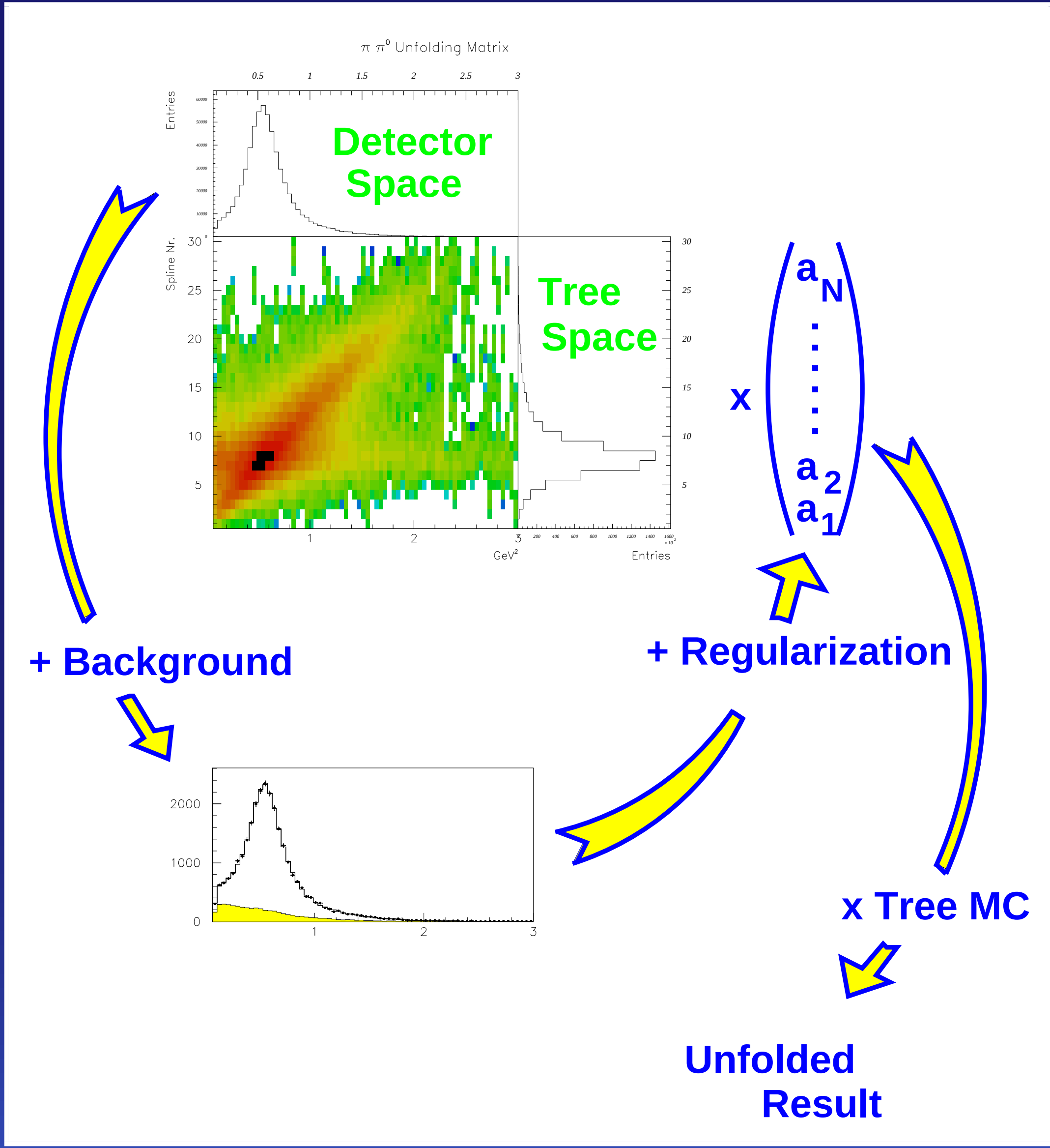
$$
\begin{aligned}
B_{\pi} &= (11.06 \pm 0.11)\,\% && (\mathrm{A}) \\
B_{\pi\pi^0} &= (25.42 \pm 0.14)\,\% && (\mathrm{V}) \\
B_{3\pi} &= (\ 9.15 \pm 0.10)\,\% && (\mathrm{A}) \\
B_{\pi 2\pi^0} &= (\ 9.17 \pm 0.14)\,\% && (\mathrm{A}) \\
B_{3\pi\pi^0} &= (\ 4.25 \pm 0.09)\,\% && (\mathrm{V}) \\
B_{\pi 3\pi^0} &= (\ 1.08 \pm 0.10)\,\% && (\mathrm{V}) \\
B_{3\pi 2\pi^0} &= (\ 0.54 \pm 0.04)\,\% && (\mathrm{A}) \\
\ldots & \\
B_{\mathrm{A}} &= (30.35 \pm 0.22)\,\% && (\mathrm{A}) \\
B_{\mathrm{V}} &= (31.61 \pm 0.23)\,\% && (\mathrm{V}) \\
B_{\mathrm{strange}} &= (\ 2.918 \pm 0.08)\,\% &&
\end{aligned}
$$

- including the leptonic Br & lifetime results:

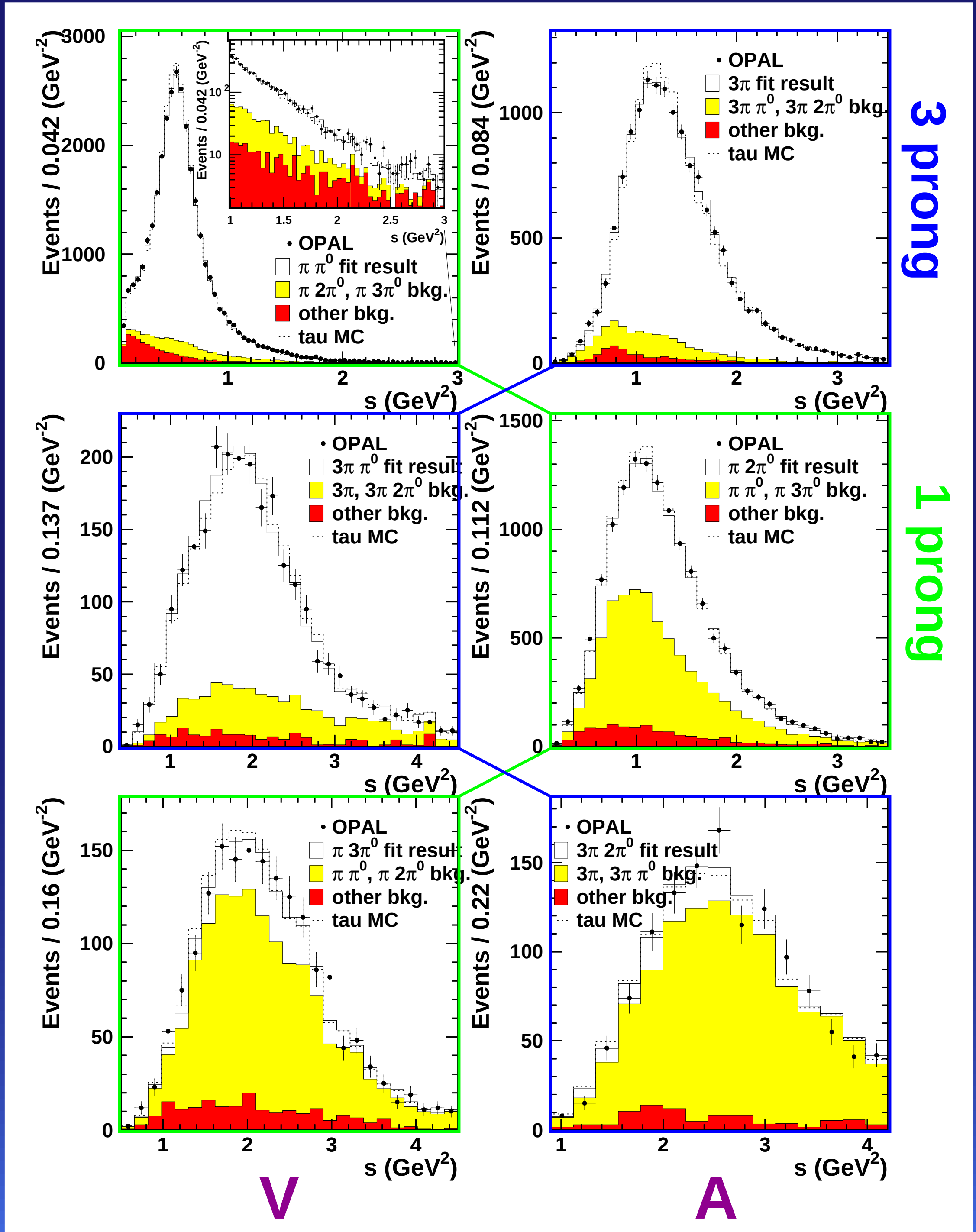
$$R_{\tau}^{\mathrm{V+A}} = 3.472 \pm 0.012$$

Measurements ► Spectral Functions

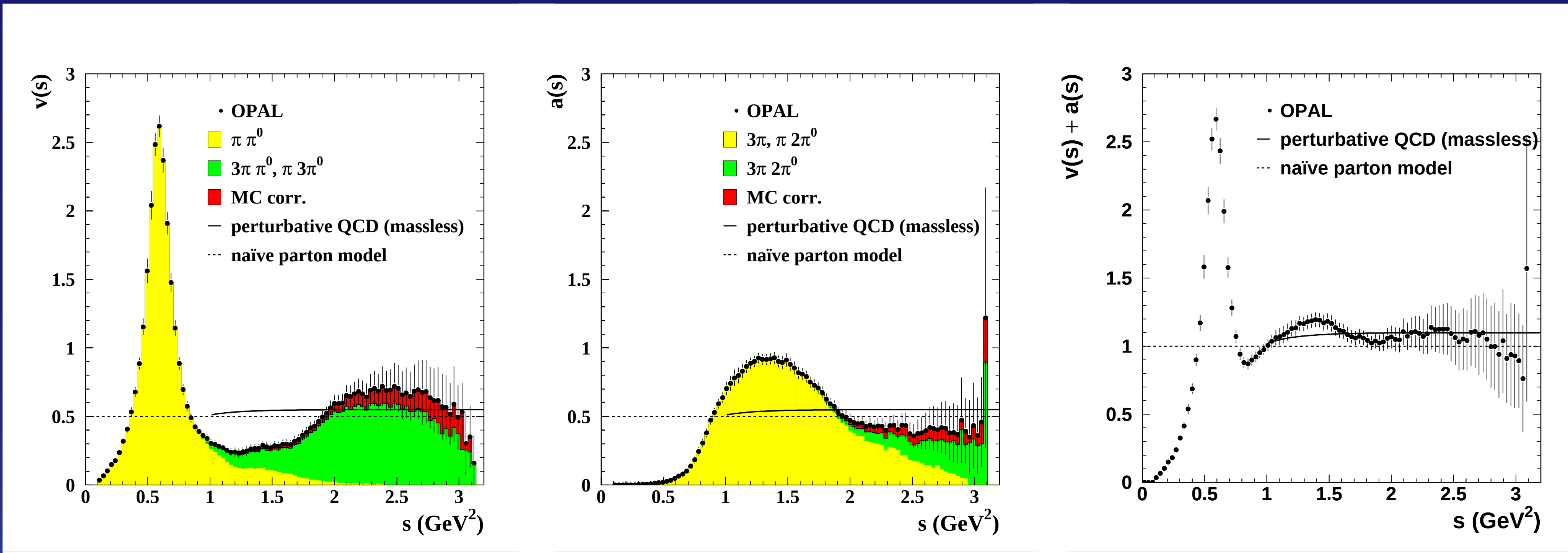
- experimental challenge is the unfolding from detector effects (measured mass $\neq$ hadron mass) and cross-feed of the signal modes (e.g. $\pi\pi^0$ background in the $\pi 2\pi^0$ channel)
- figure to the right shows unfolding principle
  - A detector response matrix is multiplied with a MC spectrum modified by a (regularized) spline function to account for deviations and added to the predicted background to fit the observed data spectrum
  - this is done for all 6 signal channels simultaneously allowing for modifications in the background shape from cross-feeding signal channls



- green (blue) boxes drawn around 1-prong (3-prong) modes which are unfolded simultaneously
- black points show the measured data (OPAL 98)
- yellow histograms show the correlated backgrounds from other signal channels fitted simultaneously
- red histograms show the un-correlated backgrounds
- dashed lines show τ-MC without fit
- open histograms show the fit results
- vector (axial-vector) channels on the left (right)



- ALEPH and OPAL measured the non-strange A/V spectral functions and more recently the strange spectral function
- plots below show the non-strange spectral functions (V, A, V+A)
- note the strong bin-to-bin correlations due to the unfolding procedure

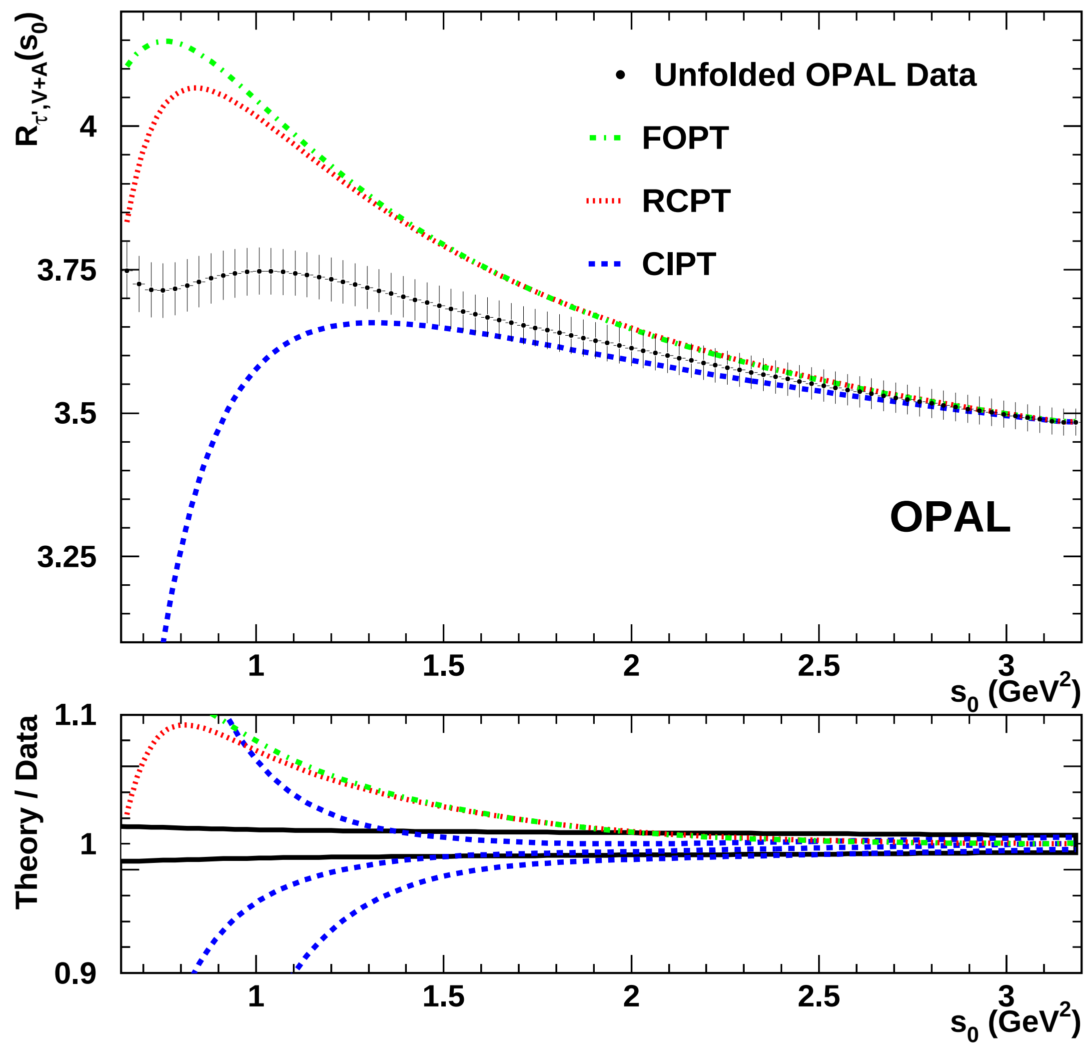


$$2\pi \mathrm{Im}\Pi^{(1)}_{V/A} = m_\tau^2 \left[6|V_{ud}|^2 \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right)\right]^{-1} \times \left(\sum_{h_{V/A}} \frac{B_{h_{V/A}}}{B_e} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \right)$$

- The combined fit of moments of $R_\tau^{V/A}$ or R_τ^{V+A} allows to establish the non-perturbative corrections
- the non-perturbative contributions of the vector (V) and axial-vector (A) part almost cancel each other in the combined (V+A) case

	V ∧ A		V+A	
Parameter	Value	Exp. Error	Value	Exp. Error
$\alpha_s(m_\tau^2)$	0.347	±0.012	0.348	±0.009
$\langle GG \rangle(\mathrm{GeV}^4)$	0.001	±0.008	-0.001	±0.012
δ_V^6	0.0256	±0.0034	–	–
δ_A^6	-0.0197	±0.0033	–	–
δ_V^8	-0.0080	±0.0013	–	–
δ_A^8	0.0041	±0.0019	–	–
δ_{V+A}^6	–	–	0.0012	±0.0056
δ_{V+A}^8	–	–	-0.0010	±0.0033

- Extrapolate the result from the fit at m_τ^2 to lower scales
  - The decay ratio of a hypothetical τ'-lepton $R_{\tau'}^{V+A}$ vs. $s_0 = m_{\tau'}^2$
- Compare with integral over measured spectral functions with adapted kinematical factor $m_\tau^2 \to m_{\tau'}^2$
- OPE describes the data well down to $\sim 1.5\,\text{GeV}^2$



- The following ingredients improved since the last measurements of ALEPH and OPAL (1998) :
  - $\Delta R_\tau^{\text{V+A}}$ reduced by a factor 2
    - Updated Branching ratios, lifetime (CLEO and LEP I)
  - ΔK_4 reduced by a factor 2
    - Partial calculations of $K_4 = 27 \pm 16$ (Baikov, Chetyrkin, and Kühn (2002))
    - I will use $\Delta K_4 = \pm 25$ instead of $\Delta K_4 = \pm 50$ which was used 1998
- New theoretical arguments
  - CIPT has smaller error than generalized FOPT and RCPT
  - The averages of generalized FOPT and RCPT agree with CIPT
- There are no new shape measurements (spectral functions)
  - Take the result for $\delta_{\text{non-pert}}^{\text{V+A}} = -0.0024 \pm 0.0025$ from CIPT fit from OPAL 1998

$$R_\tau^{\mathrm{V+A}} = 3 S_{\mathrm{EW}} |V_{ud}|^2 \left(1 + \delta'_{\mathrm{EW}} + \delta_{\mathrm{pert}} + \delta_{\mathrm{non\text{-}pert}}\right)$$

$$\begin{aligned} R_\tau^{\mathrm{V+A}} &= 3.472 \pm 0.012 \\ S_{\mathrm{EW}} &= 1.0198 \pm 0.0006 \\ V_{ud} &= 0.9745 \pm 0.0004 \\ \delta'_{\mathrm{EW}} &= 0.0010 \\ \delta_{\mathrm{non\text{-}pert}} &= -0.0024 \pm 0.0025 \end{aligned}$$

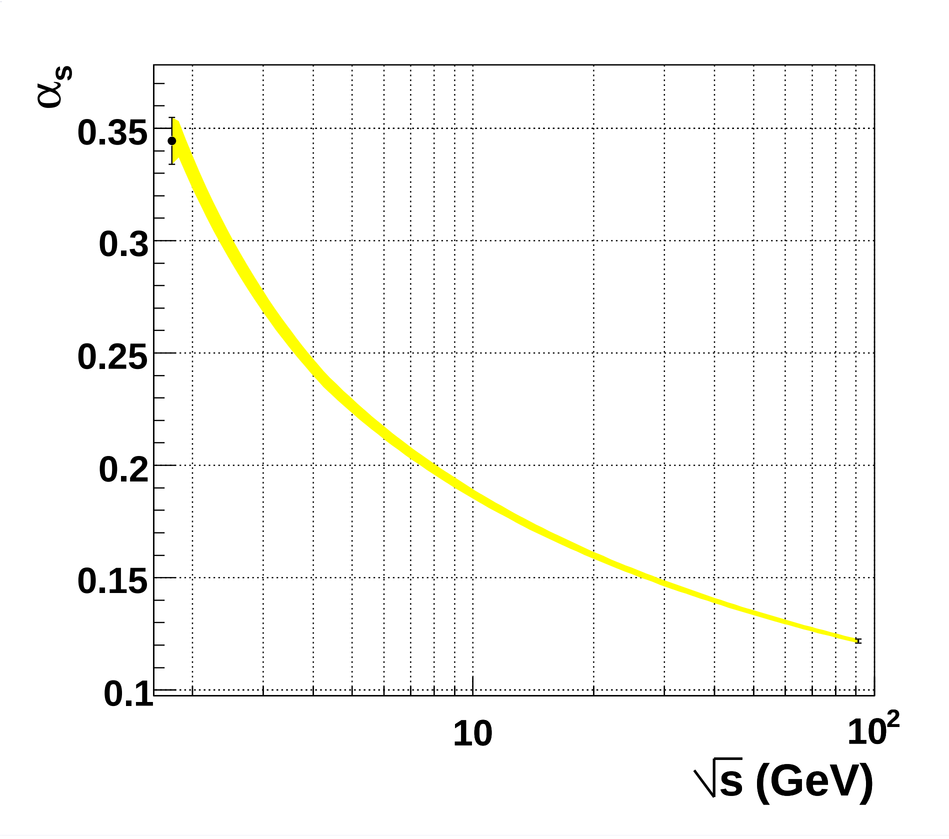
$$\delta_{\mathrm{pert}} = 0.1964 \pm 0.0041_{\mathrm{Br},\tau_\tau} \pm 0.0025_{\mathrm{non\text{-}pert}} \pm 0.0010_{V_{ud}} \pm 0.0007_{\mathrm{EW}}$$

$$\begin{aligned} K_4 &= 25 \pm 25 \\ -\mu^2/m_\tau^2 &= 1 \pm 0.6 \end{aligned}$$

$$\alpha_s(m_\tau^2) = 0.3444 \pm 0.0058_{\mathrm{exp}} \pm 0.0062_{K_4} \pm 0.0050_{\mu} \pm 0.0033_{\mathrm{non\text{-}pert}}$$

$$\alpha_s(m_\tau^2) = 0.3444 \pm 0.0058_{\mathrm{exp}} \pm 0.0086_{\mathrm{theo}}$$

- The β-function can be used to evolve this result to the Z^0-mass for comparison to other α_s measurements
- The relative uncertainty of α_s shrinks like α_s itself after evolution
  - that is the main reason why measurements at low mass-scales give smaller errors

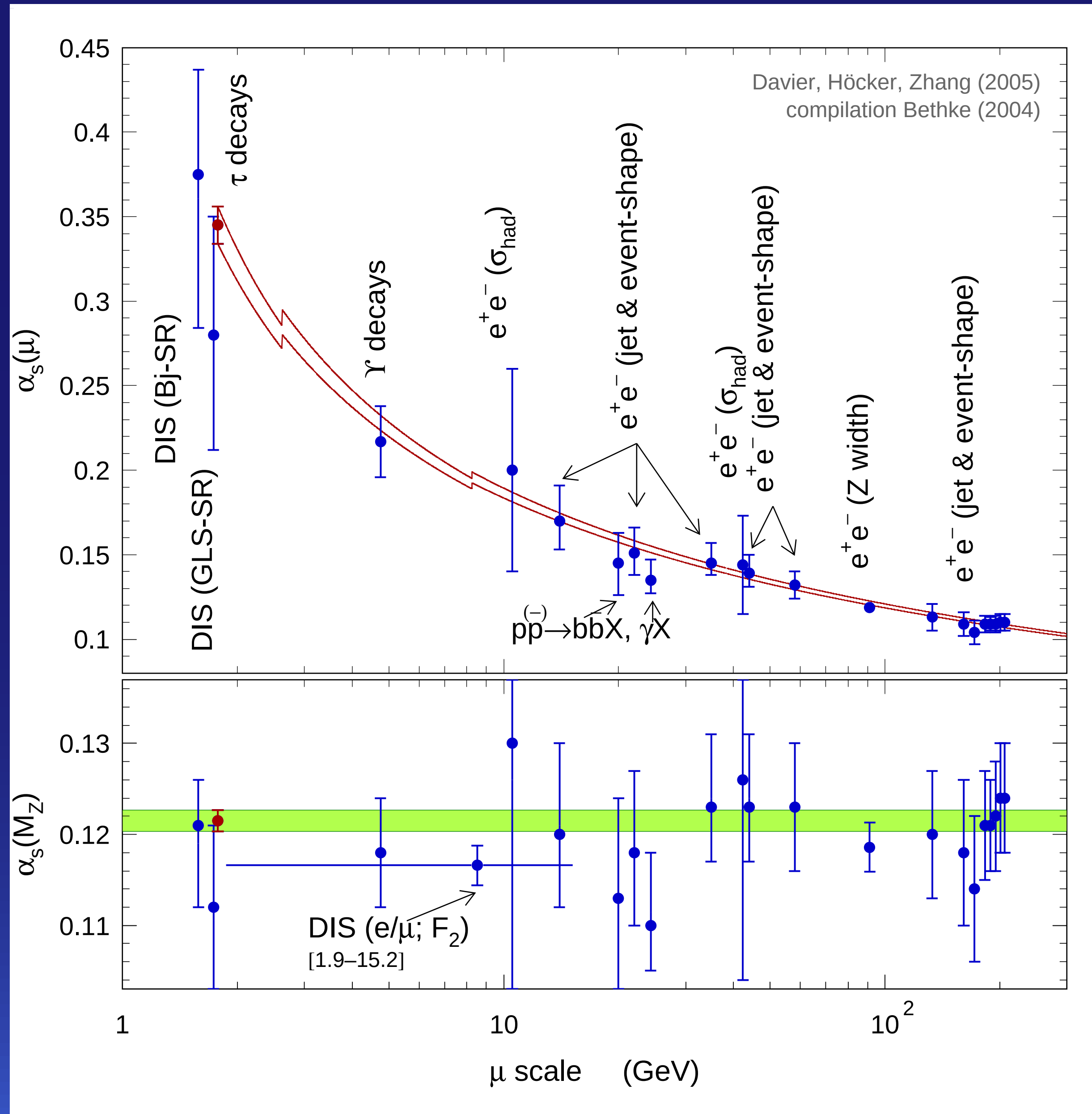


- Evolution principle:
  - $\alpha_s(m_\tau^2)^{(n_f=3)} \to \alpha_s(m_\tau^2)^{(n_f=4)}$
  - $\alpha_s(m_\tau^2)^{(n_f=4)} \to \alpha_s(m_b^2)^{(n_f=4)}$
  - $\alpha_s(m_b^2)^{(n_f=4)} \to \alpha_s(m_b^2)^{(n_f=5)}$
  - $\alpha_s(m_b^2)^{(n_f=5)} \to \alpha_s(m_Z^2)^{(n_f=5)}$
  - Variation of thresholds and quark-masses gives evolution error

$$\alpha_s(m_Z^2) = 0.12153 \pm 0.00065_{\mathrm{exp}} \pm 0.00097_{\mathrm{theo}} \pm 0.00030_{\mathrm{evol}}$$

QCD Fits ► $\alpha_s(m_\tau^2)$ ► Comparison of α_s Measurements

- From full fit to the moments and ALEPH spectral functions by Davier, Höcker, Zhang, hep-ph/0507078 one gets $\alpha_s(m_\tau^2) = 0.345 \pm 0.004_{\text{exp}} \pm 0.009_{\text{theo}}$
- compare with α_s compilation by Bethke, Nucl. Phys. (Proc. Suppl.) 135 (2004) 354.



Conclusions

- Hadronic τ-decays lead to one of the most precise measurements of α_s
  - progress in both experimental and theoretical input
  - total uncertainty still dominated by theory
- Many other important QCD tests can be done with τ-decays
  - CVC tests from comparisons to $e^+e^- \rightarrow h^{I=1}$ data
  - Chiral sum rule tests from integrals over (weighted) differences of vector and axial-vector spectral functions
  - Running of α_s below the τ-mass
  - Freezing of α_s for $s \rightarrow 0$?

:-)