EFFECT OF PROTON BUNCH PARAMETER VARIATION ON AWAKE

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Abstract

We show that the phase of the wakefields as the CERN SPS proton bunch experiences the self-modulation instability is very weakly dependent on variations of the bunch parameters by ±5%. There is a $\approx \lambda_{pe}/4$-wide region of the wakefields that remain accelerating and focusing for an electron witness bunch after the instability has grown and saturated, that is after ~ 4 m into the plasma with AWAKE base-line parameters. These results suggest that deterministic injection and acceleration of an electron witness bunch into these wakefields resulting from the self-modulation instability is, in principle, possible experimentally.

INTRODUCTION

Sending relativistic charged particle bunches through a plasma generates wakefields in the transverse and longitudinal direction with frequency defined by the electron plasma density n_e, frequency $\omega_{pe} = \left(n_e e^2/\epsilon_0 m_e\right)^{1/2}$ and wavelength $\lambda_{pe} = 2\pi c/\omega_{pe}$ [?]. In the linear regime, the transverse wakefields are $\frac{\pi}{2}$ out of phase with the longitudinal ones, so there is a region within $0.25\lambda_{pe}$ where the fields are both accelerating and focusing for an electron (or positron) bunch to be externally injected and accelerated over a long distance.

AWAKE is a proof-of-principle experiment which will be propagating the SPS (Super Proton Synchrotron) bunch through a plasma column of 10 m to drive wakefields and accelerate an externally injected electron bunch. With a baseline electron plasma density of $7{\times}10^{14}$ cm^{-3}, $\lambda_{pe} \approx$ 1.2 mm. This is many times shorter than the longitudinal rms length of the bunch (≅12 cm), which causes protons along the bunch to be locally either focused or defocused by the wakefield. This is the self-modulation instability (SMI) [?]. The focused protons form micro-bunches separated by $\approx \lambda_{pe}$, which resonantly drive the wakefields to large amplitudes.

The SMI growth and the associated evolution of the proton bunch cause the wakefield phase-velocity to change with respect to the initial bunch velocity, until eventually stabilizing at the speed of the bunch. This can be seen in Fig. ??, where we see that the on-axis field E_z moves backwards within a set window of $\xi = z - ct$ of the proton bunch as it propagates within the plasma. Near 4 m, the phase stabilizes after SMI development (i.e., lines become vertical on Fig. ??), which becomes a suitable location to inject particles. Note that the transverse wakefields follow a similar evolution (not shown).

Many simulations have been performed for AWAKE using parameters of the CERN SPS bunch. However, from an experimental viewpoint we are interested in determining how variations in the parameters of the proton bunch affect

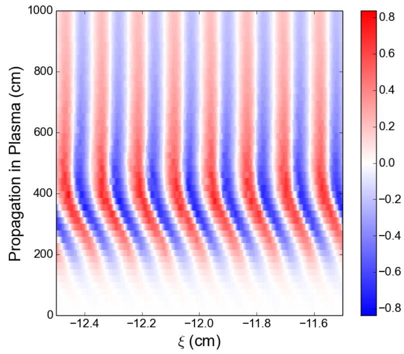

Figure 1: E_z as a function of ξ and propagation distance of the proton bunch along the plasma in a region near $\xi = -12$ cm from the proton bunch front.

the phase change at positions ξ along the bunch, i.e., where the electrons are injected. We use a variation of ±5% of the bunch parameters for this study in order to obtain the trends of the wakefields phase variation. Since all particles are relativistic and there is essentially no dephasing between them, injected electrons stay in the proper phase (accelerating and focusing) all along the acceleration process, unless the relative phase of the wakefields change.

In the experiment, the proton bunch will be co-propagating with a laser pulse at its center as it goes through rubidium (Rb) vapor. The ~ 100 fs laser pulse ionizes the Rb, creating a relativistically moving ionization front in the bunch which seeds the SMI. For the simulations, the bunch density is cut to include the sharp start of the beam/plasma interaction and for $-\sqrt{2\pi}\sigma_{zb} < \xi < 0$ is given by :

$$n_b(\xi) = n_{b0} \times 0.5 \left[1 + cos\left(\sqrt{\frac{\pi}{2}}\frac{\xi}{\sigma_{zb}}\right)\right] \times e^{-\frac{r^2}{2\sigma_{rb}{}^2}}.$$

Here $n_{b0} = N_b/[(2\pi)^{3/2}\sigma_{rb}^2\sigma_{zb}]$. Since the wakefields amplitude is proportional to n_b, the evolution of the wakefields (amplitude and phase) may be sensitive to its initial value n_{b0}, which itself depends on the bunch population N_b, and rms radius and length, σ_{rb} and σ_{zb}, respectively. These are parameters that may vary from event to event in the experiment.

For these studies, we use the particle-in-cell code OSIRIS [?] developed at IST in Lisbon and at UCLA. For these 2D simulations we use a box size of 1.61 mm with 425 grid points in r and 299.89 mm with 18000 grid points in z. The number of plasma and beam particles are 6×10^6 and 5×10^5 respectively. The simulation time step is $0.012\omega_{pe}^{-1}$. The beam and plasma parameters are the AWAKE baseline parameters: $\sigma_{rb} = 0.2$ mm, $\sigma_{zb} = 12.6$ cm, $N_b = 1.5 \times 10^{11}$,

$\epsilon_N = 3.6$ mm−mrad and the plasma density is $7 \times 10^{14}\ cm^{-3}$. It should be noted that the longitudinal rms bunch length is actually 11.4 cm, though it is linearly proportional to σ_{zb} from Eq. (**??**), therefore percentage variations affect both equivalently.

BUNCH POPULATION VARIATIONS

Simulations were run with the initial proton bunch parameters, and $N_b \pm 5\%$. The wakefield phase was analyzed as a function of bunch propagation for a range of ξ values. The relative phase of the wakefields is always calculated within the same window in ξ. It is obtained from a $cos\left(\omega_{pe}\xi/c + \phi(\xi)\right)$ fit to the E_z field in the simulation window (e.g., see Fig. **??**). The $\xi = -12$ cm region is chosen because it is there that the wakefields reach their peak value along the bunch after total propagation within the plasma, and the location of optimal wakefield phase stabilization after SMI development (see below). Figure **??** shows the relative phase shift of the wakefield for the case of the initial parameters and $N_b \pm 5\%$. We see that an increase (decrease) in N_b leads to a larger (smaller) shift backwards in ξ of the field.

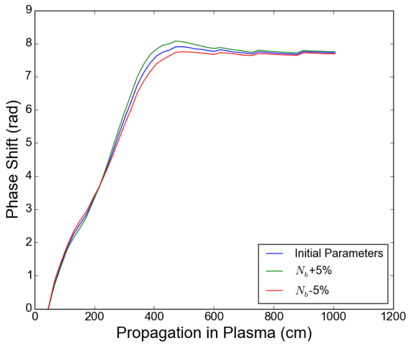

Figure 2: Phase of E_z for simulations with initial and $N_b \pm 5\%$ parameters.

We also see that the phase difference between $N_b \pm 5\%$ and the initial case is largest at regions of 4-5 m, after which it decreases and the three phases become essentially indistinguishable. This is also true at different values of ξ (not shown), though in general the phase differences are larger for larger $|\xi|$. The phase difference corresponds to about $0.03\lambda_{pe}$. This difference is small and allows in principle for placing the electron bunch near the peak of accelerating field, without the risk of loosing it to due to phase variations of the wakefields from event to event.

To find the injection point along the plasma, we calculate the energy gain for particles injected at various ξ values in one of the quarter-periods of the wakefields shown on Fig. **??**. We start at the propagation length of 1000 cm and integrate eE_z backwards along the plasma until an electron would reach a decelerating or defocusing region. The location along the plasma where the integration is stopped is shown in Fig. **??** and corresponds to the minimum injection point along the plasma. Figure **??** shows that 4 m is a good injection point, and that variations of $N_b \pm 5\%$ do not greatly affect the optimal region in ξ for injection. The injection region is $\approx 0.21\lambda_{pe}$ in ξ, smaller than the expected $\lambda_{pe}/4$ due mostly to the still-shifting wakefield phase after z = 4 m.

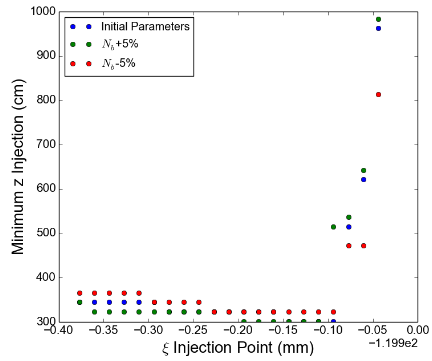

Figure 3: Minimum injection point of electrons so they remain in focusing/accelerating fields until $z = 10$ m (injection $-12.03 < \xi < -11.99$ cm)

Figure **??** shows the energy gain for injection at z = 4 m. We see the energy gain is higher when further back in ξ because this is where the peak $|E_z|$ is located. However, we see that N_b-5% case results in overall higher energy gain compared to the initial case; whereas we would expect N_b+5% to have higher energy as $E_z \propto n_{b0} \propto N_b$. This is because N_b-5% has its wakefield shifted further forward in ξ than the others, meaning an electron is closer to peak $|E_Z|$ for this case, resulting in the higher energy gain. We also see the energy gain difference from the initial case increases as we move further from peak $|E_z|$, which gives additional reason to inject at peak accelerating fields.

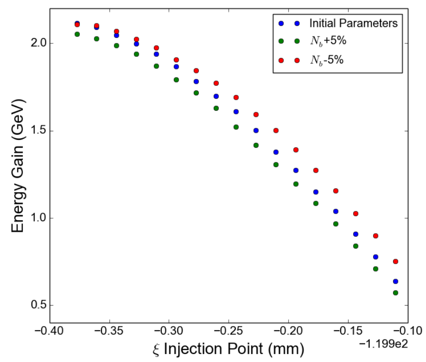

Figure 4: Energy gain of injected electrons for $z > 4$ m as a function of ξ in the optimal region for initial and $N_b \pm 5\%$ runs (injection $-12.03 < \xi < -11.99$ cm).

BUNCH SIZES VARIATIONS

We also ran simulations with $\sigma_b \pm 5\%$ (σ_{zb} and σ_{rb}). For both cases, an increase (decrease) in σ_b moves the wakefield phase forward (backwards) in ξ relative to the initial case. These effects are opposite to the variations of N_b. This implies that the wakefield phase difference is correlated with $n_b \propto \frac{N_b}{\sigma_{zb}\sigma_{rb}^2}$.

The phase differences from these simulations in the range of z = 4-5 m are lower than in the N_b case. In the neighboring regions of $\xi = -12$ cm, the differences are less than $0.03\lambda_{pe}$, implying that changes in σ_b have less of an effect than changes in N_b. Of the two, the lowest phase difference was from $\sigma_{rb}\pm5\%$.

The minimum injection point was also not significantly impacted, with the optimal region of continuous acceleration/focusing being $0.21\lambda_{pe}$ for variations of σ_{zb} and $0.22\lambda_{pe}$ for variations of σ_{rb}. The higher energy gains come from σ_b+5%, in contrast to the N_b case, also due to forward shifts in ξ of the wakefields.

OVERALL EFFECTS

With the same methods as presented above, we now examine the injection range (in terms of λ_{pe} fraction) and its dependence on the location of injection along the bunch (previously around $\xi \approx \sigma_{zb} \approx -12$ cm). In Fig. **??**, we show the injection range (as in Fig. **??**) versus injection position along the bunch for injection at z = 4 m along the plasma. Three curves are shown for each parameter change of ±5% and for each parameter variation the narrowest range value is plotted. We see that the best region for injection considering the variations is near $\xi = -11.5$ cm , which is near the rms longitudinal length of -11.4 cm.

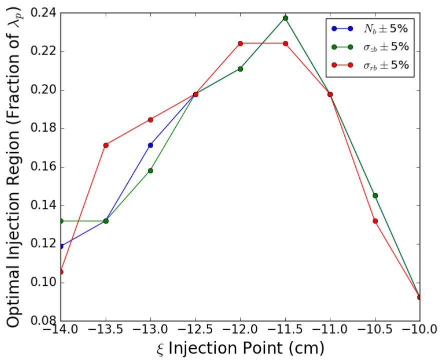

Figure 5: Size of optimal region, in fraction of λ_{pe}, for an electron injected at z=4 m for different locations in ξ.

In Figure **??** we plot the upper and lower bounds of maximum energy gain obtained for each variation, again versus the injection point along the bunch (with injection at z = 4 m along the plasma). We see also that the best region for injection is still near $\xi = -11.5$ cm due to highest energy gain and smallest energy gain interval, though the interval is largest for $\sigma_{rb}\pm5\%$ despite it having the smallest phase difference. These figures show that the best injection region in ξ at around σ_{zb} remains unchanged by bunch parameter variations of ±5%.

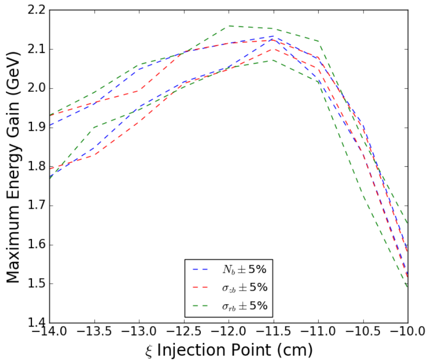

Figure 6: Upper/lower bounds of maximum energy gain of electron injected at z = 4 m for different locations in ξ.

SUMMARY AND CONCLUSION

We found that variations of ±5% in the initial proton bunch parameters (N_b, σ_{rb}, σ_{zb}) do change the phase of E_z along ξ. However, these phase differences are very small when compared to the nominal bunch parameters and are less than $0.03\lambda_{pe}$ when injecting near ξ = -12 cm along the bunch and 4 m into the plasma. This variation is well within the region of focusing/acceleration of about $0.25\lambda_{pe}$, potentially allowing for the witness bunch to be placed very close to the peak accelerating field. We also found that the wakefield phase shift is correlated with the density of the proton bunch, $n_b \propto \frac{N_b}{\sigma_{zb}\sigma_{rb}^2}$. When n_{b0} is increased, the phase of the wakefield shifts further back in ξ, whereas when it is decreased, the phase shifts further forward.

Simulations also showed the injection range for electrons at 4 m into the plasma is reduced from $\lambda_{pe}/4$ to $\approx 0.22\lambda_{pe}$ due to the wakefields slight shift in phase after SMI development, though changes in n_{b0} itself does not notably impact this range. The energy gain of electrons within this optimal injection region changes less significantly for the variations in the bunch parameters considered here for injections close to the peak accelerating field and for injection along the bunch near σ_{zb}. We also found that the best injection point along the bunch is near the rms bunch length of -11.4 cm, regardless of the studied parameters.

These results suggest that variations in the incoming bunch parameters from event to event are not a significant impediment for deterministic injection of a witness electron bunch into wakefields driven by a proton bunch that experiences the SMI.

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